

Light-Front Higher-Spin Theories in Flat Space

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Abstract

We revisit the problem of interactions of higher-spin fields in flat space. We argue that all no-go theorems can be avoided by the light-cone approach, which results in more interaction vertices as compared to the usual covariant approaches. It is stressed that there exist two-derivative gravitational couplings of higher-spin fields. We show that some reincarnation of the equivalence principle still holds for higher-spin fields — the strength of gravitational interaction does not depend on spin. Moreover, it follows from the results by Metsaev that there exists a complete chiral higher-spin theory in four dimensions. We give a simple derivation of this theory and show that the four-point scattering amplitude vanishes. Also, we reconstruct the quartic vertex of the scalar field in the unitary higher-spin theory, which turns out to be perturbatively local.

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1 Introduction

Since the early days of quantum field theory there have been many no-go results that prevent non-trivial interacting theories with massless higher-spin fields to exist. Notable examples are the Weinberg low energy theorem [1] and the Coleman-Mandula theorem [2]. One possible way out is to switch on the cosmological constant [3–5], which simultaneously avoids the no-go theorems that are formulated for QFT in flat space. Higher-spin theories in anti-de Sitter space later received a solid ground on the base of AdS/CFT correspondence [6–8] where higher-spin theories are supposed to be generic duals of free CFT’s [9–12] with certain interacting ones accessible via an alternate choice [13] of boundary conditions [9, 11, 12, 14, 15].

The fate of higher-spin theories in flat space is still unclear and is a source of controversy. The no-go theorems are still true. Also, within the local field theory approach one immediately faces certain obstructions: Aragone-Deser argument forbids minimal gravitational interactions of massless higher-spin fields [16, 17] and, even if relaxing this assumption, it is still impossible to deform the gauge algebra [18, 19]. These results are based on the gauge invariant and manifestly Lorentz covariant field description in terms of Fronsdal fields [20], which suggests another possible way out.

Indeed, gauge symmetry can be thought of as just a redundancy of description, though it turns out to be exceptionally useful in many cases. Therefore, in order to look for higher-spin theories in flat space it can be useful to turn to methods that deal with physical degrees of freedom only and thereby avoid any problems that originate from specific field descriptions. One such method is the light-cone approach, which still allows one to have a local field theory.

It is in the light-cone approach that the first examples of non-trivial cubic interactions between higher-spin fields were found in [21–23]. The covariant results followed soon after [24, 25]. A detailed classification of cubic vertices within the light cone approach is now available in all dimensions for massive and massless fields of arbitrary spin and symmetry type [26–29].

In this paper we revisit the problem of constructing higher-spin theories in flat space, specifically in four-dimensions. First of all, we argue that at least formally the most powerful no-go theorems are avoided by the light-cone approach. Also, we recall that there is a mismatch between the covariant cubic vertices and those found in [21–23] by the light-cone methods: there exist exceptional vertices not seen by some of the covariant methods. In particular, there does exist a two-derivative gravitational vertex for a field of any spin [30–32], which is also evident in the language of amplitudes [33, 34].

Having the gravitational higher-spin vertex at our disposal we prove that fields of any spin couple to gravity universally, i.e. some form of the equivalence principle is still true for higher-spin fields. In fact, the strength of the gravitational coupling does not depend on spin at all.

A remarkable result obtained by Metsaev in [35, 36] is that one can fix the cubic vertex without having to perform the full quartic analysis. We present a simple derivation of this result, which clarifies the assumptions. Based on this solution, we note that there exists a consistent non-trivial higher-spin theory in flat space. This theory contains graviton, massless higher-spin fields, the two-derivative gravitational vertices as well as other vertices. The action terminates at cubic vertices. Like in the self-dual Yang-Mills theory the four-point scattering amplitude vanishes. The only feature is that it breaks parity and is non-unitary. Nevertheless, it provides a counterexample to a widespread belief that higher-spin theories in flat space do not exist at all.

Aiming at the unitary and parity preserving higher-spin theory in flat space we reconstruct the part of the quartic Hamiltonian that contains self-interactions of the scalar field, which can be regarded as the flat space counterpart of the AdS_4 result [37, 38].

The outline is as follows. In Section 2 we discuss how to avoid the famous no-go results. In Section 3 we review the basics of the light-cone approach with the main result being the classification of all possible couplings that was obtained in [21–23, 35, 36]. The relation to the Lorentz covariant

classification is spelled out in Section 3.2. In Section 4 we present a complete chiral higher-spin theory with the details of the derivation of the Metsaev solution [35, 36] devoted to Appendix C. The scalar part of the quartic Hamiltonian of the unitary higher-spin theory is reconstructed in Section 5. The higher-spin equivalence principle is derived in Section 6. We conclude with some discussion of possible extensions of these results in Section 7.

2 Avoiding No-Go Theorems

In the distant past it was a common belief that higher-spin theories, i.e. the theories with massless fields with spin greater than two, are not consistent. The most notable examples of such no-go theorems are Weinberg low energy theorem [1], Coleman-Mandula theorem [2] and the Aragone-Deser argument [16]. We briefly discuss them below, see also a very nice review [39], as to point out how all of them can be avoided.

Our conclusion is that there are still good chances to have nontrivial higher-spin theories in flat space. Moreover, we will present an example of consistent chiral theory in Section 4. However, it should be stressed that while higher-spin theories may avoid the assumptions of the no-go theorems they may not defy the spirit of these theorems: there are strong indications that S -matrix should be trivial in some sense. For example, for the case of conformal higher-spin theories the S -matrix is a combination of $\delta(s, t, u)$ [32, 40, 41] and the AdS/CFT duals of unbroken higher-spin theories must be free CFT's [42–46], which should be thought of as examples of trivial holographic S -matrices.

Weinberg low energy theorem. A serious restriction comes from the Weinberg low energy theorem [1] that eventually leads to too many conservation laws, when massless higher-spin fields are present. As a result of checking linearized gauge invariance or Lorentz invariance of the n -particle amplitude with one soft spin- s particle attached one finds

$$\begin{array}{ccc}
 \begin{array}{c} \diagup \\ \bullet \\ \diagdown \end{array} & \Longleftrightarrow & \sum_i g_s^i p_{\mu_1}^i \dots p_{\mu_{s-1}}^i = 0 \quad (2.1)
 \end{array}$$

where g_s^i is the coupling constant of the i -th species to a spin- s field. For $s = 1$ one discovers that the total (electric) charge is conserved. For $s = 2$ one finds a linear combination of momenta weighted by g_2^i whose clash with the momentum conservation law $\sum_i p_\mu^i = 0$ can only be resolved by the equivalence principle, i.e. all fields must couple to gravity universally, $g_2^i = \text{const}$.

For the higher-spin case $s > 2$ one finds too many conservations laws, which is a rank $(s - 1)$ tensorial expression, with the only solution given by permutations of momenta at the condition that all coupling constants are the same.

In the course of the proof of the theorem one makes an explicit use of Lorentz covariant vertices. In particular, the expressions are manifestly Lorentz covariant. This is not the case in the light-cone approach where the vertices do not have a manifestly Lorentz covariant form. It would be interesting

to reconsider the Weinberg theorem as to see whether these assumptions can be weakened.¹

Coleman-Mandula theorem. The famous Coleman-Mandula theorem [2] prevents S -matrix from having symmetry generators, beyond those of the Poincare group, that transform under the Lorentz group. Under assumptions of non-triviality of the symmetry action, discrete mass spectrum and the analyticity of the S -matrix in Mandelstam invariants, it can be shown that the symmetry algebra can only be a product of the Poincare group and a group of internal symmetries whose generators are Lorentz scalars. It does not apply to the case of $d = 1 + 1$ QFT, where only forward/backward scattering is possible, so S -matrix must have scattering angles $\theta = 0, \pi$ and thereby it is not analytic. The essence of the proof is that the scattering process is a map from one set of momenta to another one and the momenta are restricted by energy-momentum conservation, which is a Lorentz vector equation. Existence of some other charges that transform non-trivially under the spacetime symmetry would impose tensorial equations on momenta, e.g. like in Weinberg theorem, which would restrict possible processes to exchanges of momenta like in $1 + 1$ or trivialize the scattering completely. One way the original Coleman-Mandula theorem can be avoided is by assuming that symmetry generators transform as spinors, which leads to supersymmetry.

One of the assumptions of the theorem is to have a finite number of particles below any mass-shell. This is certainly not true in higher-spin theories where the spectrum should contain infinitely many massless particles [25, 44, 47, 48]. It would be interesting to weaken the assumptions of the theorem [49].

Aragone-Deser argument/No canonical gravity coupling. Contrary to the Weinberg and Coleman-Mandula theorems, this argument is local and is attached to specific field variables [16, 17]. It says that the canonical way of putting fields on a curved background by replacing partial derivatives with covariant ones does not work for massless higher-spin fields. Indeed, in checking the gauge invariance of the action we have to commute derivatives, which brings the Riemann tensor:

$$S = \int \nabla \phi \nabla \phi + \dots, \quad \delta \phi = \nabla \xi, \quad \delta S = \int (\phi \dots) (\nabla \cdot \xi R_{\bullet\bullet\bullet\bullet} + \xi \nabla \cdot R_{\bullet\bullet\bullet\bullet}). \quad (2.2)$$

Unlike low-spin examples, we find the full four-index Riemann tensor — the structure that cannot be compensated by any modifications of the action/gauge transformations. For $s = 1$ the action is manifestly gauge invariant, while for $s = 3/2$ we find not the full Riemann tensor but its trace, the Ricci tensor, which allows to overcome the problem by going to supergravities.

The argument above makes use of the specific field variables and of the manifestly Lorentz covariant methods. Obviously, this is avoided by the light-cone approach. We will emphasize in Section 3.2 that there exists in fact a two-derivative gravitational coupling of massless higher-spin fields to gravity [21–23], which is not captured by covariant studies [50–52].

¹We are grateful to Sasha Zhiboedov for the useful discussion of this problem.

BCFW. A relatively new no-go type result came from the BCFW approach [33, 34, 53–56]. However, higher-spin theories are clearly different from Yang-Mills theory and even gravity and are not expected to have an S -matrix that is analytic. Moreover, BCFW approach is essentially based on the assumption of certain behavior of amplitudes for infinite BCFW shifts. It is not a priori clear whether these assumptions can be justified in the higher-spin case. Some works towards weakening these assumptions include [33, 34, 53–55].

Three dimensions. Massless higher-spin fields do not have local degrees of freedom in three-dimensions [57–60] and therefore the no-go theorems discussed above do not apply, see [61, 62] and references thereon for more detail.

AdS. Another option to avoid the no-go theorems is to simply abandon the flat space and go to anti-de Sitter background [3–5] since the no-go theorems discussed above were formulated for QFT’s in flat space.

3 Living on Light-Front

In this Section we review the light-cone approach to relativistic dynamics. Next, we discuss the classification of cubic vertices that results from the light-cone dynamics and confront it with the covariant methods. The main lesson is that there are more vertices in the light-cone approach. In particular there are two-derivative interaction vertices $s - s - 2$ of a spin- s field and a graviton, which can be called gravitational. The reader not interested in the somewhat boring details² can jump directly to Section 3.2. It is worth stressing that the Yang-Mills theory, when rewritten in the light-cone approach, is a theory of scalar fields in the adjoint of the global symmetry group. Similarly, gravity is a theory of two scalar fields with no symmetries like diffeomorphisms whatsoever.

3.1 Basics

Quantum field theory in flat space in its most rigorous definition requires a Hilbert space endowed with the unitary action of the Poincare algebra, i.e. the generators of Lorentz transformations J^{AB} and translations P^A should be realized as to obey:³

$$[P^A, P^B] = 0, \tag{3.1}$$

$$[J^{AB}, P^C] = P^A \eta^{BC} - P^B \eta^{AC}, \tag{3.2}$$

$$[J^{AB}, J^{CD}] = J^{AD} \eta^{BC} - J^{BD} \eta^{AC} - J^{AC} \eta^{BD} + J^{BC} \eta^{AD}. \tag{3.3}$$

In free theory the generators are quadratic in the quantum fields and have to receive certain corrections when interactions are switched on.

²Nice, pedagogical exposition of the light-cone approach can also be found in [28, 63].

³It is convenient to choose the mostly plus convention for η_{AB} and $A, B, \dots = 0, \dots, d-1$.

Canonical quantization begins with postulating the canonical commutation relations of fields and momenta at some fixed time, which encodes the choice of the Cauchy surface for evolution. As was pointed out by Dirac [64] there are different quantization schemes depending on the choice of the quantization surface.⁴ The difference is in the stability group that preserves the surface. The generators associated with the stability group, called *kinematical*, do not receive any quantum corrections and stay quadratic in the fields on the Cauchy surface. The left-over generators, called *dynamical*, do deform.

For the canonical equal time choice $t = t_0$ the stability subgroup of the Poincare group $ISO(3, 1)$ ($ISO(d - 1, 1)$ in d dimensions) consists of spacial rotations and translations, while boosts and time translations $P_0 = H$ do not preserve the surface. Therefore, there are four generators (d in the case of d dimensions) that receive corrections due to interactions.

The light-front is the light-like quantization surface. The canonical choice is $x^+ = 0$, so that x^+ is treated as the time direction and $H = P^-$ is the Hamiltonian.⁵ As a result only $(d - 1)$ generators need to be deformed, which is the least number possible. A somewhat unfortunate feature of any non-covariant quantization, including the light-cone one, is that due to the manifest Lorentz symmetry breaking we have to deal with many more generators whose total number is the same. The ten generators of $iso(3, 1)$ can be split into kinematical (K) and dynamical (D) as follows:

$$\text{kinematical :} \quad P^+, P^a, J^{a+}, J^{+-}, J^{ab} \quad : 7 \quad (3.4)$$

$$\text{dynamical :} \quad P^-, J^{a-} \quad : 3 \quad (3.5)$$

The time evolution of any operator G is determined by the Hamiltonian $\dot{G} = i[H, G]$. Therefore, if the Poincare algebra relations are satisfied at the initial light-cone time $x^+ = 0$, then they will be satisfied at all times. This has a useful consequence that some of the generators having explicit x^+ dependence

$$J^{-+} = x^- \partial^+ - x^+ P^-, \quad J^{a+} = x^a \partial^+ - x^+ \partial^a, \quad (3.6)$$

should be declared to be kinematical, as we did above, since the dynamical part vanishes at $x^+ = 0$. The x^+ -dependence can then be reconstructed by virtue of the equations of motion.

Let us now list the commutation relations and the consequences thereof. A generator G can be split $G = G_2 + G_{\text{int}}$ into its free part G_2 and an interacting part G_{int} , the latter being absent for the kinematical generators. The kinematical generators are fixed once and for all times. As for dynamical generators the procedure is that there are commutators that simply constrain the dynamical generators to have certain dependence on the kinematical variables. Also, there are few other relations that represent nontrivial equations to be solved for the dynamical generators.

⁴Let us note that there are several different things that bear almost the same name: light-cone gauge, light-front (or light-cone) quantization and one can also combine the two by quantizing a theory on a light-front with the light-cone gauge imposed.

⁵In the light-front coordinates $A = +, -, a$, etc., $\eta^{+-} = \eta^{-+} = 1$ and $\eta^{ab} = \delta^{ab}$. Also, in $4d$ one can replace $x^{1,2}$ with two complex conjugate variables z, \bar{z} , so that the metric is $2x^+x^- + 2z\bar{z}$.

$[\mathbf{K}, \mathbf{K}] = \mathbf{K}$. The kinematical generators do not receive any corrections, so this part stays unchanged and is of no use, which is why we list them here-below for completeness:

$$[P^+, P^b] = 0, \quad [P^a, P^b] = 0, \quad (3.7a)$$

$$[J^{-+}, P^+] = -P^+, \quad [J^{-+}, P^c] = 0, \quad [J^{a+}, P^+] = 0, \quad (3.7b)$$

$$[J^{a+}, P^c] = -P^+ \delta^{ac}, \quad [J^{ab}, P^+] = 0, \quad [J^{ab}, P^c] = P^a \delta^{bc} - P^b \delta^{ac}, \quad (3.7c)$$

$$[J^{-+}, J^{c+}] = -J^{c+}, \quad [J^{-+}, J^{cd}] = 0, \quad [J^{a+}, J^{c+}] = 0, \quad (3.7d)$$

$$[J^{a+}, J^{cd}] = \eta^{ac} J^{d+} - \eta^{ad} J^{c+}, \quad [J^{ab}, J^{cd}] = \text{as usual}. \quad (3.7e)$$

$[\mathbf{K}, \mathbf{D}] = \mathbf{K}$. This set of relations splits into two parts. First one is $[K, D] = 0$ -type relations that immediately restrict the dynamical generators. The second one are $[K, D] = K$ -type commutators, which imply that the interacting part of D commutes to the given K , i.e. $[K, D_{\text{int}}] = 0$, which is due to $K_{\text{int}} = 0$ and the right-hand side being taken into account by free fields, $[K_2, D_2] = K_2$.

$$[P^+, P^-] = 0, \quad [P^a, P^-] = 0, \quad [J^{a+}, P^-] = P^a, \quad (3.8a)$$

$$[J^{ab}, P^-] = 0, \quad [J^{a+}, J^{c-}] = \delta^{ac} J^{-+} - J^{ac}, \quad [J^{a-}, P^+] = P^a. \quad (3.8b)$$

$[\mathbf{K}, \mathbf{D}] = \mathbf{D}$. These relations are similar to the previous ones and constrain the dynamical generators to behave nicely under the light-front symmetries:

$$[J^{-+}, P^-] = P^-, \quad [J^{-+}, J^{c-}] = J^{c-}, \quad [J^{ab}, J^{c-}] = J^{a-} \delta^{bc} - J^{b-} \delta^{ac}, \quad (3.9a)$$

$$[J^{a-}, P^c] = -P^- \delta^{ac}. \quad (3.9b)$$

$[\mathbf{D}, \mathbf{D}] = \mathbf{0}$. This class consists of the actual equations to be solved and constitutes the main problem of the light-cone approach:

$$[J^{a-}, J^{c-}] = 0, \quad [J^{a-}, P^-] = 0. \quad (3.10)$$

Summary. There are three dynamical generators: two boosts J^{a-} and the Hamiltonian $H = P^-$:

$$H = P^- = H_2 + H_{\text{int}}, \quad J^{a-} = J_2^{a-} + x^a H_{\text{int}} + J_{\text{int}}^{a-}, \quad (3.11)$$

where we split them into the free and interacting parts and moreover symbolically extract the dependence of J^{a-} on H_{int} . The commutation relations imply that H_{int} is a centralizer of several kinematical generators:

$$[H_{\text{int}}, T] = 0 : \quad T = P^+, P^a, J^{a+}, J^{ab}. \quad (3.12)$$

Initially, J^{a-} commutes only to P^+ , J^{a+} . The shift by $x^a H_{\text{int}}$ cancels H in $[J^{a-}, P^c] = -P^c \delta^{ac}$, which becomes the $[K, D] = 0$ -type relation for J_{int}^{a-} . Therefore, we find

$$[J_{\text{int}}^{a-}, T] = 0 : \quad T = P^+, P^a, J^{a+}. \quad (3.13)$$

The $[K, D] = D$ -type relations, when written for the deformations, give

$$[J^{-+}, H_{\text{int}}] = H_{\text{int}}, \quad [J^{-+}, J_{\text{int}}^{c-}] = J_{\text{int}}^{c-}, \quad (3.14)$$

$$[J^{ab}, J_{\text{int}}^{c-}] = J_{\text{int}}^{a-} \delta^{bc} - J_{\text{int}}^{b-} \delta^{ac}. \quad (3.15)$$

All the constraints above can be explicitly solved and one is left with (3.10), of which only $[H, J^{a-}] = 0$ needs to be solved, as we explain below.

3.1.1 Free Field Realization

We have just discussed which commutation relations need to be solved. Further progress can only be made for specific theories. The general comment is that the quantization on the light-front leads to second-class constraints.⁶ Indeed, the kinetic term $\frac{1}{2}(\partial\phi)^2$, when written in the light-cone coordinates, $\partial^+ \phi \partial^- \phi + \frac{1}{2}(\nabla\phi)^2$, is linear in the velocity $\partial^- \phi$ and hence the momenta, i.e. the primary constraints, cannot be solved for $\partial^- \phi$. Therefore, the bracket is the Dirac bracket.

From now on we confine ourselves to live in the four-dimensional world. The nice feature of the $4d$ world is that all massless spinning particles have two degrees of freedom, i.e. made of two scalar fields except for the spin-zero particle, which equals one scalar field. A spin- s particle has two states with helicities $\pm s$ and can be described as two fields $\Phi^{\pm s}(x)$ that are complex conjugate. It is convenient to work with the fields that are Fourier transformed with respect to x^- and transverse coordinates x^a :

$$\Phi(x, x^+) = (2\pi)^{-\frac{d-1}{2}} \int e^{+i(x^- p^+ + x \cdot p)} \Phi(p, x^+) d^{d-1} p, \quad (3.16)$$

$$\Phi(p, x^+) = (2\pi)^{-\frac{d-1}{2}} \int e^{-i(x^- p^+ + p \cdot x)} \Phi(x, x^+) d^{d-1} x. \quad (3.17)$$

In the $4d$ world the equal time commutation relations that follow from the Dirac bracket are:

$$[\Phi^\mu(p, x^+), \Phi^\lambda(q, x^+)] = \delta^{\mu, -\lambda} \frac{\delta^3(p+q)}{2p^+}. \quad (3.18)$$

From now on we set $x^+ = 0$ and will omit the arguments in most of the cases. It is very easy to find

⁶See very nice books [65, 66] for quantization of field theories with constraints.

the kinematical generators of the Poincare algebra in the Fourier space:⁷

$$\hat{P}^+ = \beta, \quad \hat{P} = p, \quad \hat{\bar{P}} = \bar{p}, \quad (3.19a)$$

$$\hat{J}^{z+} = -\beta \frac{\partial}{\partial \bar{p}}, \quad \hat{J}^{\bar{z}+} = -\beta \frac{\partial}{\partial p}, \quad \hat{J}^{-+} = -N_\beta - 1 = -\frac{\partial}{\partial \beta} \beta, \quad (3.19b)$$

$$\hat{J}^{z\bar{z}} = N_p - N_{\bar{p}} - \lambda, \quad (3.19c)$$

where $N_p = p\partial_p$ is the Euler operator, *idem.* for $N_{\bar{p}}$, N_β and we sometimes use $\partial_\beta = \partial/\partial\beta$, etc. The generators are supposed to act on $\Phi^\lambda \equiv \Phi_p^\lambda \equiv \Phi^\lambda(\beta, p, \bar{p}, x^+ = 0)$. The dynamical generators at the free level are:

$$H_2 = -\frac{p\bar{p}}{\beta}, \quad \begin{aligned} \hat{J}_2^{z-} &= \frac{\partial}{\partial \bar{p}} \frac{p\bar{p}}{\beta} + p \frac{\partial}{\partial \beta} + \lambda \frac{p}{\beta}, \\ \hat{J}_2^{\bar{z}-} &= \frac{\partial}{\partial p} \frac{p\bar{p}}{\beta} + \bar{p} \frac{\partial}{\partial \beta} - \lambda \frac{\bar{p}}{\beta}. \end{aligned} \quad (3.20)$$

The Poincare charges can be built in a standard way:

$$Q_\xi = \int p^+ d^3p \Phi_{-p}^{-\mu} O_\xi(p, \partial_p) \Phi_p^\mu, \quad (3.21)$$

where O_ξ is the generator of the Poincare algebra associated with a Killing vector ξ . We draw reader's attention to the fact that the integration measure is p^+ . The Poincare algebra is then realized via commutators

$$\delta_\xi \Phi^\mu(p, x^+) = [\Phi^\mu(p, x^+), Q_\xi]. \quad (3.22)$$

Due to the nontrivial integration measure the conjugate operators are defined as

$$O^\dagger = -\frac{1}{p^+} O^T(-p) p^+, \quad (3.23)$$

where the transposed operator is defined via integration by parts as usual, e.g. $p^T = p$, $\partial_p^T = -\partial_p$. The generators of the Poincare algebra given above are Hermitian, $O^\dagger = O$. In particular, we find $p^\dagger = p$. With the help of (3.18) and

$$\delta_\xi \Phi^\mu(p, x^+) = \frac{1}{2} O_\xi(p, \partial_p) \Phi^\mu(p, x^+) + \frac{1}{2} O_\xi^\dagger \Phi^\mu(p, x^+) = O_\xi(p) \Phi_p^\mu \quad (3.24)$$

one can verify all the commutation relations:

$$[Q_\xi, Q_\eta] = Q_{[\xi, \eta]}, \quad [\delta_\xi, \delta_\eta] \Phi = +\delta_{[\xi, \eta]} \Phi. \quad (3.25)$$

⁷Following the light-cone commandments we rename $\beta = p^+$ otherwise the paper will not be understandable at all.

(3.24) follows from a more general formula for the action of Q on an arbitrary functional $F[\Phi]$:

$$[F(\Phi), Q_\xi] = \int dv O_\xi(v) \Phi_v^\nu \frac{\partial}{\partial \Phi_v^\nu} F[\Phi], \quad (3.26)$$

which we will immediately apply to read off the constraints imposed by kinematical generators on the dynamical ones.

3.1.2 Kinematical Constraints

An appropriate ansatz for the Hamiltonian H and dynamical boosts J^{a-} reads:⁸

$$H = H_2 + \sum_n \int d^{3n} q \delta\left(\sum q_i\right) h_{\lambda_1, \dots, \lambda_n}^{q_1, \dots, q_n} \Phi_{q_1}^{\lambda_1} \dots \Phi_{q_n}^{\lambda_n}, \quad (3.27a)$$

$$J^{z-} = J_2^{z-} + \sum_n \int d^{3n} q \delta\left(\sum q_i\right) \left[j_{\lambda_1, \dots, \lambda_n}^{q_1, \dots, q_n} - \frac{1}{n} h_{\lambda_1, \dots, \lambda_n}^{q_1, \dots, q_n} \left(\sum_j \frac{\partial}{\partial \bar{q}_j} \right) \right] \Phi_{q_1}^{\lambda_1} \dots \Phi_{q_n}^{\lambda_n}, \quad (3.27b)$$

$$J^{\bar{z}-} = J_2^{\bar{z}-} + \sum_n \int d^{3n} q \delta\left(\sum q_i\right) \left[\bar{j}_{\lambda_1, \dots, \lambda_n}^{q_1, \dots, q_n} - \frac{1}{n} h_{\lambda_1, \dots, \lambda_n}^{q_1, \dots, q_n} \left(\sum_j \frac{\partial}{\partial q_j} \right) \right] \Phi_{q_1}^{\lambda_1} \dots \Phi_{q_n}^{\lambda_n}, \quad (3.27c)$$

where the delta function imposes the conservation of the total q^+ and transverse momenta q, \bar{q} , which is a consequence of the translation invariance imposed by P^a and P^+ , (3.12), (3.13). The rest of the kinematical generators imposes the following constraints:

$$J^{a+} : \quad \left(\sum_k \beta_k \frac{\partial}{\partial q_k^a} \right) h_{\lambda_1, \dots, \lambda_n}^{q_1, \dots, q_n} \sim 0, \quad (3.28a)$$

$$J^{a+} : \quad \left(\sum_k \beta_k \frac{\partial}{\partial q_k^a} \right) \bar{j}_{\lambda_1, \dots, \lambda_n}^{q_1, \dots, q_n} \sim 0, \quad \text{same for } \bar{j}, \quad (3.28b)$$

$$J^{z\bar{z}} : \quad \left[\sum_k (N_{q_k} - N_{\bar{q}_k}) + \sum \lambda_k \right] h_{\lambda_1, \dots, \lambda_n}^{q_1, \dots, q_n} \sim 0, \quad (3.28c)$$

$$J^{-+} : \quad \sum_k \beta_k \frac{\partial}{\partial \beta_k} h_{\lambda_1, \dots, \lambda_n}^{q_1, \dots, q_n} \sim 0, \quad (3.28d)$$

$$J^{-+} : \quad \sum_k \beta_k \frac{\partial}{\partial \beta_k} \bar{j}_{\lambda_1, \dots, \lambda_n}^{q_1, \dots, q_n} \sim 0, \quad \text{same for } \bar{j}, \quad (3.28e)$$

$$J^{z\bar{z}} : \quad \left[\sum_k (N_{q_k} - N_{\bar{q}_k}) + \sum \lambda_k - 1 \right] j_{\lambda_1, \dots, \lambda_n}^{q_1, \dots, q_n} \sim 0, \quad (3.28f)$$

$$J^{z\bar{z}} : \quad \left[\sum_k (N_{q_k} - N_{\bar{q}_k}) + \sum \lambda_k + 1 \right] \bar{j}_{\lambda_1, \dots, \lambda_n}^{q_1, \dots, q_n} \sim 0, \quad (3.28g)$$

where ~ 0 means an equality up to an overall delta-function $\delta^{d-1}(\sum q_k)$.

In practice it is tedious to keep all delta-functions unresolved and it is more convenient to choose

⁸The derivatives can also act both on h and wave functions, which is equivalent to redefining j^{a-} .

some independent momenta as basic variables. Moreover, (3.28a)-(3.28b) imply that everything depends on specific combinations of momenta \mathbb{P}_{km} :

$$J^{a+} : \quad \sum_k \beta_k \frac{\partial}{\partial q_k^a} \sim 0 \quad \implies \quad \mathbb{P}_{km}^a = q_k^a \beta_m - q_m^a \beta_k. \quad (3.29)$$

There are $N - 2$ such independent variables for N -point function. In the $4d$ case we have

$$\mathbb{P}_{km} = q_k \beta_m - q_m \beta_k, \quad \bar{\mathbb{P}}_{km} = \bar{q}_k \beta_m - \bar{q}_m \beta_k. \quad (3.30)$$

Therefore, we assume that some $N - 2$ variables out of all \mathbb{P} 's have been chosen and

$$h_{\lambda_1 \dots \lambda_n}(q_1, \dots, q_n) = h_{\lambda_1 \dots \lambda_n}(\mathbb{P}_{km}, \bar{\mathbb{P}}_{km}, \beta_k), \quad (3.31)$$

$$j_{\lambda_1 \dots \lambda_n}(q_1, \dots, q_n) = j_{\lambda_1 \dots \lambda_n}(\mathbb{P}_{km}, \bar{\mathbb{P}}_{km}, \beta_k), \quad \text{same for } \bar{j}. \quad (3.32)$$

The rest of the system of kinematical constraints can be rewritten as

$$J^{z\bar{z}} : \quad \left[\mathbb{P} \frac{\partial}{\partial \mathbb{P}} - \bar{\mathbb{P}} \frac{\partial}{\partial \bar{\mathbb{P}}} + \sum \lambda_k \right] h_{\lambda_1, \dots, \lambda_n}^{q_1, \dots, q_n} \sim 0, \quad (3.33a)$$

$$J^{-+} : \quad \left[\mathbb{P} \frac{\partial}{\partial \mathbb{P}} + \bar{\mathbb{P}} \frac{\partial}{\partial \bar{\mathbb{P}}} + \sum_k \beta_k \frac{\partial}{\partial \beta_k} \right] h_{\lambda_1, \dots, \lambda_n}^{q_1, \dots, q_n} \sim 0, \quad (3.33b)$$

$$J^{-+} : \quad \left[\mathbb{P} \frac{\partial}{\partial \mathbb{P}} + \bar{\mathbb{P}} \frac{\partial}{\partial \bar{\mathbb{P}}} + \sum_k \beta_k \frac{\partial}{\partial \beta_k} \right] j_{\lambda_1, \dots, \lambda_n}^{q_1, \dots, q_n} \sim 0, \quad (3.33c)$$

$$J^{-+} : \quad \left[\mathbb{P} \frac{\partial}{\partial \mathbb{P}} + \bar{\mathbb{P}} \frac{\partial}{\partial \bar{\mathbb{P}}} + \sum_k \beta_k \frac{\partial}{\partial \beta_k} \right] \bar{j}_{\lambda_1, \dots, \lambda_n}^{q_1, \dots, q_n} \sim 0, \quad (3.33d)$$

$$J^{z\bar{z}} : \quad \left[\mathbb{P} \frac{\partial}{\partial \mathbb{P}} - \bar{\mathbb{P}} \frac{\partial}{\partial \bar{\mathbb{P}}} + \sum \lambda_k - 1 \right] j_{\lambda_1, \dots, \lambda_n}^{q_1, \dots, q_n} \sim 0, \quad (3.33e)$$

$$J^{z\bar{z}} : \quad \left[\mathbb{P} \frac{\partial}{\partial \mathbb{P}} - \bar{\mathbb{P}} \frac{\partial}{\partial \bar{\mathbb{P}}} + \sum \lambda_k + 1 \right] \bar{j}_{\lambda_1, \dots, \lambda_n}^{q_1, \dots, q_n} \sim 0. \quad (3.33f)$$

The above conditions are very simple homogeneity constraints and need no further comments.

3.1.3 Cubic Vertices

The first nontrivial dynamical constraints arise at the cubic order. First of all, the kinematics of three $(d - 1)$ -dimensional momenta restricted by the conservation delta-function is very simple. There is one independent \mathbb{P}^a variable since $\mathbb{P}_{12}^a = \mathbb{P}_{23}^a = \mathbb{P}_{31}^a$. Therefore, in $4d$ we have just \mathbb{P} and $\bar{\mathbb{P}}$. It is advantageous to represent it in a manifestly cyclic-invariant way:

$$\mathbb{P}_{12}^a = \dots = \mathbb{P}^a = \frac{1}{3} [(\beta_1 - \beta_2)q_3^a + (\beta_2 - \beta_3)q_1^a + (\beta_3 - \beta_1)q_2^a], \quad (3.34)$$

$$\sigma_{123}\mathbb{P} = \mathbb{P}, \quad \sigma_{12}\mathbb{P} = \sigma_{23}\mathbb{P} = \sigma_{13}\mathbb{P} = -\mathbb{P}. \quad (3.35)$$

Therefore, \mathbb{P} belongs to the totally anti-symmetric representation of S_3 . There is an identity that is of utter importance for the cubic approximation:

$$\sum_j \frac{\partial}{\partial q_j} \mathbb{P} = 0. \quad (3.36)$$

Also, at the three-point level we find

$$\sum_i H_2(q_i) = \frac{\mathbb{P}\bar{\mathbb{P}}}{\beta_1\beta_2\beta_3} = \frac{\mathbb{P} \cdot \mathbb{P}}{2\beta_1\beta_2\beta_3}. \quad (3.37)$$

Now we proceed to the dynamical constraints. The first one is $[H, J^{a-}] = 0$ restricted to the cubic order in fields Φ :

$$[H, J^{a-}] \Big|_3 = [H_3, J_2^{a-}] - [J_3^{a-}, H_2] = 0, \quad (3.38)$$

which, after using the magic identity (3.36), can be shown to lead to

$$\sum_i H_2(q_i) j_3 = \sum_i (\hat{J}_2^{z-})^T h_3, \quad \sum_i H_2(q_i) \bar{j}_3 = \sum_i (\hat{J}_2^{\bar{z}-})^T \bar{h}_3, \quad (3.39)$$

where the transposed generators are

$$(\hat{J}_2^{z-})^T = -\frac{q\bar{q}}{\beta} \frac{\partial}{\partial \bar{q}} - q \frac{\partial}{\partial \beta} + \lambda \frac{q}{\beta}, \quad (\hat{J}_2^{\bar{z}-})^T = -\frac{q\bar{q}}{\beta} \frac{\partial}{\partial q} - \bar{q} \frac{\partial}{\partial \beta} - \lambda \frac{\bar{q}}{\beta}. \quad (3.40)$$

Now one can make an appropriate ansatz for h_3 that solves the kinematical constraints (3.33), act with J_2^T and read off j_3 and \bar{j}_3 up to possible redefinitions. The most general case is studied in Appendix A, while below we simply quote the representation given by Metsaev in [35, 36]. The first results on cubic interactions of HS fields were obtained in [21–23] in a slightly different base.

At the interaction level there is always a problem of fixing the field redefinitions. The light-cone approach is not free of this ambiguity too. At the cubic order redefinitions allow one to eliminate powers of $\mathbb{P}\bar{\mathbb{P}} \sim H_2$, but not each of the two separately. Therefore, the most natural choice of the redefinition frame is to have purely holomorphic vertices. It is worth stressing that this is not the most natural choice in the covariant approaches. The vertices are [35, 36]:

$$h_{\lambda_1, \lambda_2, \lambda_3} = C^{\lambda_1, \lambda_2, \lambda_3} \frac{\bar{\mathbb{P}}^{\lambda_1 + \lambda_2 + \lambda_3}}{\beta_1^{\lambda_1} \beta_2^{\lambda_2} \beta_3^{\lambda_3}} + \bar{C}^{-\lambda_1, -\lambda_2, -\lambda_3} \frac{\mathbb{P}^{-\lambda_1 - \lambda_2 - \lambda_3}}{\beta_1^{-\lambda_1} \beta_2^{-\lambda_2} \beta_3^{-\lambda_3}}, \quad (3.41a)$$

$$j_{\lambda_1, \lambda_2, \lambda_3} = +\frac{2}{3} C^{+\lambda_1, +\lambda_2, +\lambda_3} \frac{\bar{\mathbb{P}}^{+\lambda_1 + \lambda_2 + \lambda_3 - 1}}{\beta_1^{+\lambda_1} \beta_2^{+\lambda_2} \beta_3^{+\lambda_3}} \Lambda^{\lambda_1, \lambda_2, \lambda_3}, \quad (3.41b)$$

$$\bar{j}_{\lambda_1, \lambda_2, \lambda_3} = -\frac{2}{3} \bar{C}^{-\lambda_1, -\lambda_2, -\lambda_3} \frac{\mathbb{P}^{-\lambda_1 - \lambda_2 - \lambda_3 - 1}}{\beta_1^{-\lambda_1} \beta_2^{-\lambda_2} \beta_3^{-\lambda_3}} \Lambda^{\lambda_1, \lambda_2, \lambda_3}, \quad (3.41c)$$

where

$$\Lambda = \beta_1(\lambda_2 - \lambda_3) + \beta_2(\lambda_3 - \lambda_1) + \beta_3(\lambda_1 - \lambda_2). \quad (3.42)$$

Here $C^{\lambda_1, \lambda_2, \lambda_3}$ and $\bar{C}^{-\lambda_1, -\lambda_2, -\lambda_3}$ are two sets of coupling constants which are a priori independent. For dimensional reasons we have to introduce a parameter l_P with the dimension of length as to compensate for the higher powers of momenta, as was noted as early as [21–23]:

$$C^{\lambda_1, \lambda_2, \lambda_3} = (l_P)^{\lambda_1 + \lambda_2 + \lambda_3 - 1} c^{\lambda_1, \lambda_2, \lambda_3}, \quad \text{same for } \bar{C}. \quad (3.43)$$

In higher-spin theories the parameter will be naturally associated with the Planck length as the Einstein-Hilbert vertex is a part of the set above and corresponds to $C^{2,2,-2}$.

The light-cone locality implies that the powers of \mathbb{P} , $\bar{\mathbb{P}}$ must be non-negative or whenever $\sum \lambda_i = 0$ we should have $\lambda_i = 0$. The latter is due to the fact that j_3^{a-} has one power of \mathbb{P} or $\bar{\mathbb{P}}$ less. The exception is when all $\lambda_i = 0$, which is the scalar self-interaction vertex, since it leads to $j_3 = 0$, which is implied in (3.41).

Let us stress that the light-cone approach deals only with physical degrees of freedom, so the light-cone gauge is a unitary gauge, but it is not an on-shell method. Nevertheless, there is a striking relation between the on-shell amplitude methods and the light-cone approach [67–69]. One can introduce

$$|i\rangle = \frac{2^{1/4}}{\sqrt{\beta_i}} \begin{pmatrix} \bar{q}_i \\ -\beta_i \end{pmatrix} = 2^{1/4} \begin{pmatrix} \bar{q}_i \beta_i^{-1/2} \\ -\beta_i^{1/2} \end{pmatrix}, \quad (3.44)$$

so that the basic building blocks of cubic vertices can be found in

$$[i|j] = \sqrt{\frac{2}{\beta_i \beta_j}} \bar{\mathbb{P}}_{ij}, \quad (3.45)$$

and analogously one can define $|i\rangle$. As a result, the cubic vertices, i.e. Hamiltonian density h_3 , can be rewritten in a more suggestive form:

$$C^{s_1, s_2, s_3} [12]^{s_1 + s_2 - s_3} [23]^{s_2 + s_3 - s_1} [13]^{s_1 + s_3 - s_2} + c.c., \quad (3.46)$$

which are the usual amplitudes for three helicity fields [33, 34].

3.2 Light-Cone vs. Covariant Vertices

On one hand, the general formula for cubic vertices (3.41) is given above. On another hand, the classification of cubic vertices in covariant approaches is also available.⁹ Remarkably, by confronting

⁹There is a vast literature on cubic vertices in covariant approaches. We give a minimalistic list of references [3, 18, 19, 51–53, 70–73] that allows one to trace all the initial results and further developments by following references therein/thereon, the accent being put on the diversity of approaches. For our purposes it is sufficient to confront the classification of the light-cone vertices [27–29] with some of the covariant results [18, 52].

the light-cone vertices and covariant ones we observe a mismatch in the number of local interactions, see also [31–34], which is due to the difference between locality in light-cone and covariant approaches.

In the light-cone approach, the vertices can be arranged by the number of derivatives for a given triplet of spins $s_1 - s_2 - s_3$. The power counting is easy in the light-cone approach too: one counts the total power of the transverse momenta, or of \mathbb{P}^a , which is the same. Therefore, vertices (3.41) have $|\lambda_1 + \lambda_2 + \lambda_3|$ derivatives, where we note that the helicities can be negative. In covariant approaches one can distinguish between the following classes of vertices, though this classification is incomplete:

Current Interaction. The $C^{0,0,s}$ vertex has s derivatives and corresponds to the usual current interaction where a spin- s current $\phi_0 \partial^s \phi_0$ built of two scalar fields is contracted with the Fronsdal field ϕ_s . This is the simplest vertex that involves one higher-spin field and for $s = 1$ corresponds to the current interaction while for $s = 2$ to the coupling $T_{\mu\nu} g^{\mu\nu}$ of the stress-tensor to gravity.

Non-abelian Vertices. For every spin the vertex $C^{s,s,-s}$ has s derivatives and drives the non-abelian deformation of the gauge algebra in the covariant approach [18]. In $d > 4$ there can be more than one non-abelian self-interaction, but in $4d$ this seems to be the only one. In particular, $C^{1,1,-1}$ is the Yang-Mills vertex and $C^{2,2,-2}$ is the Einstein-Hilbert vertex. Having such vertices activated is important for non-triviality of the theory. There is a covariant vertex $s - s - 2$ with $(2s - 2)$ derivatives that can be called gravitational, but it certainly cannot result from $\partial \rightarrow \nabla$ replacement in the action due to its higher derivative nature for $s > 2$.

Abelian Vertices. It is also possible to construct the $(s_1 + s_2 + s_3)$ -derivative vertex C^{s_1,s_2,s_3} . It does not induce any deformation of the gauge algebra and therefore cannot be used as a seed of any interesting theory, while such vertices can be required for consistency at the quartic order. This indeed happens for higher-spin theory, but does not happen for Yang-Mills and Einstein theory, where F^3 and R^3 vertices can be dropped (or have an independent coupling constant in front of them).

As a result, there is a mismatch between the covariant and the light-cone dictionaries. Indeed, $s_1 - s_2 - s_3$ vertex in $4d$ can have $s_1 + s_2 + s_3 - 2 \min(s_1, s_2, s_3)$ or $s_1 + s_2 + s_3$ derivatives [28], i.e. one can have two vertices at most. On contrary, the light-cone vertices exist for any triplet of helicities, i.e. there can be up to four independent complex vertices (3.41). When the reality condition is imposed, $C = \bar{C}$, this number still reduces to three vertices at most. For example, there exists an exceptional series of vertices $C^{s',s,-s}$, $s > s'$, that have less derivatives (transverse momenta) and is absent in covariant approaches. In particular, this exceptional series contains a two-derivative $C^{2,s,-s}$ gravitational vertex!¹⁰ The $s = 2$ case corresponds to the usual Einstein-Hilbert vertex and does not look strange anymore.

The existence of such vertex seems paradoxical in view of the simplest no-go result, known as the Aragone-Deser argument [16]. As we discussed in Section 2 the argument is explicitly Lorentz

¹⁰The existence of such an vertex was stressed in [30], though it is certainly present in [23, 27, 35, 36].

covariant and is formulated in terms of specific field content, Fronsdal fields, rather than in terms of physical degrees of freedom and therefore is avoided by the light-cone approach.

More generally, within the covariant approaches the statement that some interaction does not exist depends heavily on the field content. Few examples include: local electromagnetic interactions mediated by A_μ will look non-locally in terms of $F_{\mu\nu}$; the formulation of self-dual fields may require an infinite number of auxiliary fields [74]; there may be the need for some compensator or other auxiliary fields, see e.g. [75]; a seeming breaking of Lorentz symmetries might be needed, e.g. [76].

Another result that seems to be in tension with the existence of the two-derivative $s-s-2$ vertex is the Weinberg low energy theorem. As we discussed in Section 2, the light-cone approach seems to avoid the assumptions of the theorem.

It is worth stressing that the existence of the strange low-derivative vertex is not a unique feature of the light-cone approach and is also seen via amplitude techniques [33, 34], as (3.46) reveals.

3.3 Quartic Analysis and Beyond

The main result of the cubic approximation is the list of all possible cubic vertices that can be used for constructing any theory. As is usual for the cubic approximation, the coefficients C^{s_1, s_2, s_3} and \bar{C}^{s_1, s_2, s_3} in front of the cubic vertices are completely free and will be fixed by the quartic analysis, which we will now proceed to.

After an appropriate ansatz for j_n and h_n that solves the kinematical constraints (3.33) is chosen, one has to solve

$$[J_2^{a-}, H_n] - [H_2, J_n^{a-}] = \sum_{\substack{i, j > 2 \\ i+j=n}} [H_i, J_j^{a-}]. \quad (3.47)$$

The right-hand side contains the source that is made of the structures that are supposed to have been already found at orders lower than n . On evaluating the commutators on the left-hand side we discover an operator that commutes to the delta-functions that impose momenta conservation:

$$\tilde{J}_2^{a-}(q_i) = \tilde{J}^{a-}(q_i, \partial_{q_i}) = J_2^{a-}(q_i)^T - H_2(q_i) \frac{1}{n} \left(\sum_j \frac{\partial}{\partial q_j^a} \right). \quad (3.48)$$

For further convenience let us denote

$$\mathbf{J}_2^{a-} = \sum_i \tilde{J}_2^{a-}(q_i), \quad \mathbf{H}_2 = \sum_i H_2(q_i). \quad (3.49)$$

Now the main equation can be rewritten as

$$\mathbf{H}_2 j_n^{a-} = \mathbf{J}_2^{a-}[h_n] + \sum_{\substack{i, j > 2 \\ i+j=n}} [H_i, J_j^{a-}]. \quad (3.50)$$

It is evident that the equation can formally be solved for j_n by dividing both sides by \mathbf{H}_2 . However, when we go on-shell \mathbf{H}_2 is the sum of p^- components and will vanish. Therefore, the crucial requirement is to adjust the right-hand side of (3.50) as to make it be proportional to \mathbf{H}_2 . Then, we can safely divide by \mathbf{H}_2 and get some local j_n . If such a solution is found we can all go to the beach because it can be shown, see Appendix B, that $[J^{a-}, J^{b-}] = 0$ contains no new relations and holds true automatically.

Parity transformations. The parity is a useful symmetry as well as its breaking is. The parity transformations are defined as

$$P(\Phi^\lambda) = \Phi^{-\lambda}, \quad P(C^{\lambda_i}) = \bar{C}^{-\lambda_i}, \quad P(\mathbb{P}) = \bar{\mathbb{P}}. \quad (3.51)$$

If we are interested in the unitary higher-spin theory we have to impose $C = \bar{C}$, otherwise the Hamiltonian is complex. For the chiral theories this condition will be violated.

4 Complete Chiral Higher-Spin Theory

In this section we will show that there exists a complete chiral higher-spin theory in flat space. Also we will elaborate on the solution obtained by Metsaev in [35, 36], with the technical details placed in Appendix C.

The starting point is the ansatz for H_4 and J_4^{a-} that solves the kinematical constraints (3.33) and is free of delta-functions:

$$H_4 = \int h_4(\mathbb{P}_{12}, \mathbb{P}_{34}; \beta_i) \Phi_{q_1}^{\lambda_1} \dots \Phi_{q_4}^{\lambda_4}, \quad (4.1)$$

$$J_4 = \int j_4(\mathbb{P}_{12}, \mathbb{P}_{34}; \beta_i) \Phi_{q_1}^{\lambda_1} \dots \Phi_{q_4}^{\lambda_4} - \frac{1}{4} h_4(\mathbb{P}_{12}, \mathbb{P}_{34}; \beta_i) \left(\sum \frac{\partial}{\partial \bar{q}_j} \right) \Phi_{q_1}^{\lambda_1} \dots \Phi_{q_4}^{\lambda_4}, \quad (4.2)$$

$$\bar{J}_4 = \int \bar{j}_4(\mathbb{P}_{12}, \mathbb{P}_{34}; \beta_i) \Phi_{q_1}^{\lambda_1} \dots \Phi_{q_4}^{\lambda_4} - \frac{1}{4} h_4(\mathbb{P}_{12}, \mathbb{P}_{34}; \beta_i) \left(\sum \frac{\partial}{\partial q_j} \right) \Phi_{q_1}^{\lambda_1} \dots \Phi_{q_4}^{\lambda_4}. \quad (4.3)$$

The consistency condition to be solved at the quartic order is

$$\mathbf{H}_2 j_4^{a-} = \mathbf{J}_2^{a-}[h_4] + [H_3, J_3^{a-}]. \quad (4.4)$$

For definiteness, let us consider the component of this equation with $a = z$, i.e. the one for j while the equation for \bar{j} is similar. Following Metsaev [35, 36], we note that both $\mathbf{H}_2 j_4^{a-}$ and $\mathbf{J}_2^{a-}[h_4]$, if non-zero, are at least linear in q . On the other hand, the contribution from the anti-holomorphic part of H_3 (which we denote $H_3(\bar{\mathbb{P}})$) to $[H_3, J_3]$ is q -independent and thus has to vanish on its own:

$$[H_3(\bar{\mathbb{P}}), J_3] = 0, \quad [H_3(\mathbb{P}), \bar{J}_3] = 0. \quad (4.5)$$

Therefore, the parts of the quartic consistency condition (4.4) that have CC and $\bar{C}\bar{C}$ structure con-

stants form a system of equation that is decoupled from h_4 and j_4 ! We called these parts holomorphic. The $C\bar{C}$ -part of $[H_3, J_3^{a-}]$ does couple to h_4 and j_4^{a-} and its analysis is a real challenge:

$$\mathbf{H}_2 j_4 = \mathbf{J}_2[h_4] + [H_3(\mathbb{P}), J_3], \quad \mathbf{H}_2 \bar{j}_4 = \bar{\mathbf{J}}_2[h_4] + [H_3(\bar{\mathbb{P}}), \bar{J}_3]. \quad (4.6)$$

The holomorphic equations (4.5) impose a strong constraint on coupling constants $C^{\lambda_1, \lambda_2, \lambda_3}$ and in fact can be used to fix all of them in terms of a single coupling constant provided some reasonable conditions on the theory are imposed. Certainly one can fulfill the holomorphic constraints by selecting the abelian vertices only, which gives an infinitely many of not so interesting solutions. The complete classification of solutions is still lacking. Also, there is a series of solutions where a single spin- s field couples to graviton and Yang-Mills field, see also Section 6.2. Generally, one can find solutions if $C^{\lambda_1, \lambda_2, \lambda_3}$ is sufficiently sparse and does not include non-abelian self-interactions of higher-spin fields, $C^{s, s, -s}$, as they force one to introduce all higher-spin fields together. This is coherent with the studies [42–46, 48] of uniqueness of higher-spin symmetries in the context of AdS/CFT.

The solution found by Metsaev [35, 36] has a remarkably simple form:

$$C^{\lambda_1, \lambda_2, \lambda_3} = \frac{(l_P)^{\lambda_1 + \lambda_2 + \lambda_3 - 1}}{\Gamma(\lambda_1 + \lambda_2 + \lambda_3)}. \quad (4.7)$$

where l_P has to have a dimension of length and, since there is graviton in the multiplet, it is natural to identify l_P with the Planck length. Even though the final result (4.7) is very simple, its derivation is not obvious. To make our discussion more self-contained we present in Appendix C a relatively simple and explicit derivation of Metsaev's formula also highlighting some of its important features.

The remarkable property of the $4d$ light-cone approach is that the consistency equations split into two decoupled systems of CC and $\bar{C}\bar{C}$ equations (4.5) for the structure constants of the cubic action and an additional system (4.6) that contains h_4 and j_4^{a-} . The latter is the system to be solved for h_4 and j_4^{a-} while the source involves $C\bar{C}$. A crucial observation is that one can simply set $\bar{C} = 0$ and hence $h_4 = 0$, $j_4^{a-} = 0$ is a solution of (4.6). Obviously, setting $h_{n>3} = 0$ and $j_{n>3}^{a-} = 0$ together with C from (4.7) provides a complete solution to all orders! The only feature is that the Hamiltonian is complex and for that reason the theory is non-unitary.

For completeness we write below the full Hamiltonian of the chiral higher-spin theory and an action that can be obtained by the Legendre transform:

$$H = \int \Phi_{-q}^{-\lambda} \frac{q\bar{q}}{\beta} \Phi_q^\lambda + \int \frac{(l_P)^{\lambda_1 + \lambda_2 + \lambda_3 - 1}}{\Gamma(\lambda_1 + \lambda_2 + \lambda_3)} \frac{\bar{\mathbb{P}}^{\lambda_1 + \lambda_2 + \lambda_3}}{\beta_1^{\lambda_1} \beta_2^{\lambda_2} \beta_3^{\lambda_3}} \Phi_{q_1}^{\lambda_1} \Phi_{q_2}^{\lambda_2} \Phi_{q_3}^{\lambda_3} \delta^3(q_1 + q_2 + q_3), \quad (4.8a)$$

$$S = - \int \partial_A \Phi^{-\lambda} \partial^A \Phi^\lambda + \int \frac{(l_P)^{\lambda_1 + \lambda_2 + \lambda_3 - 1}}{\Gamma(\lambda_1 + \lambda_2 + \lambda_3)} \frac{\bar{\mathbb{P}}^{\lambda_1 + \lambda_2 + \lambda_3}}{(\partial_1^+)^{\lambda_1} (\partial_2^+)^{\lambda_2} (\partial_3^+)^{\lambda_3}} \Phi^{\lambda_1} \Phi^{\lambda_2} \Phi^{\lambda_3}, \quad (4.8b)$$

where the sum over all helicities λ is assumed and the fields in the last line $\Phi^\lambda(x)$ carry full space-time

dependence. The derivatives are to act on the corresponding fields and in the last line

$$\bar{\mathbb{P}} = \frac{1}{3} [(\partial_1^+ - \partial_2^+) \bar{\partial}_3 + (\partial_2^+ - \partial_3^+) \bar{\partial}_1 + (\partial_3^+ - \partial_1^+) \bar{\partial}_2] . \quad (4.9)$$

Tree-Level four-point Amplitude. In this paragraph we show that the four-point scattering amplitude in the chiral theory given by (4.8) vanishes. The total s -channel exchange between external fields with helicities $\lambda_1, \lambda_2, \lambda_3$ and λ_4 is

$$\mathcal{A}_s = \sum_{\omega} \frac{1}{(\lambda_1 + \lambda_2 + \omega - 1)!} \frac{\bar{\mathbb{P}}_{12}^{\lambda_1 + \lambda_2 + \omega}}{\beta_1^{\lambda_1} \beta_2^{\lambda_2}} \frac{1}{(q_1 + q_2)^2} \frac{1}{(\lambda_3 + \lambda_4 - \omega - 1)!} \frac{\bar{\mathbb{P}}_{34}^{\lambda_3 + \lambda_4 - \omega}}{\beta_3^{\lambda_3} \beta_4^{\lambda_4}} . \quad (4.10)$$

Performing summation and adding contributions from other channels we find

$$\begin{aligned} \mathcal{A} = \frac{1}{2(\Lambda - 2)! \prod_{i=1}^4 \beta_i^{\lambda_i}} & \left(\frac{\bar{\mathbb{P}}_{12} \bar{\mathbb{P}}_{34}}{(q_1 + q_2)^2} [(\bar{\mathbb{P}}_{34} - \bar{\mathbb{P}}_{12})^{\Lambda-2} - (\bar{\mathbb{P}}_{34} + \bar{\mathbb{P}}_{12})^{\Lambda-2}] \right. \\ & + \frac{\bar{\mathbb{P}}_{13} \bar{\mathbb{P}}_{24}}{(q_1 + q_3)^2} [(\bar{\mathbb{P}}_{24} - \bar{\mathbb{P}}_{13})^{\Lambda-2} - (\bar{\mathbb{P}}_{24} + \bar{\mathbb{P}}_{13})^{\Lambda-2}] \\ & \left. + \frac{\bar{\mathbb{P}}_{14} \bar{\mathbb{P}}_{23}}{(q_1 + q_4)^2} [(\bar{\mathbb{P}}_{14} - \bar{\mathbb{P}}_{23})^{\Lambda-2} - (\bar{\mathbb{P}}_{14} + \bar{\mathbb{P}}_{23})^{\Lambda-2}] \right), \end{aligned} \quad (4.11)$$

where $\Lambda = \sum_{i=1}^4 \lambda_i$. For on-shell momenta one has

$$(q_i + q_j)^2 = -\frac{2}{\beta_i \beta_j} \mathbb{P}_{ij} \bar{\mathbb{P}}_{ij}, \quad \sum_{j=1}^4 \frac{\mathbb{P}_{ij} \bar{\mathbb{P}}_{jk}}{\beta_j} = 0. \quad (4.12)$$

These identities allow one to show that

$$E \equiv \frac{\bar{\mathbb{P}}_{12} \bar{\mathbb{P}}_{34}}{(q_1 + q_2)^2} = -\frac{\bar{\mathbb{P}}_{13} \bar{\mathbb{P}}_{24}}{(q_1 + q_3)^2} = \frac{\bar{\mathbb{P}}_{14} \bar{\mathbb{P}}_{23}}{(q_1 + q_4)^2}. \quad (4.13)$$

Also, using

$$\begin{aligned} 2A &\equiv \bar{\mathbb{P}}_{12} + \bar{\mathbb{P}}_{34} = -\bar{\mathbb{P}}_{14} + \bar{\mathbb{P}}_{23}, \\ 2B &\equiv \bar{\mathbb{P}}_{13} - \bar{\mathbb{P}}_{24} = \bar{\mathbb{P}}_{34} - \bar{\mathbb{P}}_{12}, \\ 2C &\equiv \bar{\mathbb{P}}_{14} + \bar{\mathbb{P}}_{23} = -\bar{\mathbb{P}}_{13} - \bar{\mathbb{P}}_{24} \end{aligned} \quad (4.14)$$

we find

$$\mathcal{A} = \frac{E}{2(\Lambda - 2)! \prod_{i=1}^4 \beta_i^{\lambda_i}} ((2B)^{\Lambda-2} - (2A)^{\Lambda-2} - (2B)^{\Lambda-2} + (2C)^{\Lambda-2} + (2A)^{\Lambda-2} - (2C)^{\Lambda-2}) = 0.$$

Therefore, the four-point amplitude is zero and we expect all higher-point tree-level amplitudes to vanish as well.

5 Quartic Hamiltonian of Unitary Higher-spin Theory

In the previous section we considered the consistency condition (4.4), which we repeat here for convenience

$$\mathbf{H}_2 j_4^{a-} = \mathbf{J}_2^{a-}[h_4] + [H_3, J_3^{a-}], \quad (5.1)$$

for $z = a$. Following Metsaev, we showed that they contain an independent sector, which involves only the couplings $C^{\lambda_1, \lambda_2, \lambda_3}$ of the chiral part of the Hamiltonian and allow to fix them up to inessential normalisation factors, see (C.10) in Appendix C. The remaining consistency conditions can be solved trivially by setting to zero the coupling constants $\bar{C}^{-\lambda_1, -\lambda_2, -\lambda_3}$ of the conjugated Hamiltonian. This choice of the cubic Hamiltonian is consistent on its own and thus defines a complete higher-spin theory (4.8).

Analogously, one can start from (5.1) with $a = \bar{z}$ and fix the coupling constants $\bar{C}^{-\lambda_1, -\lambda_2, -\lambda_3}$ of the anti-chiral Hamiltonian. Setting to zero the remaining coupling constants one obtains a consistent anti-chiral higher-spin theory.

As is expected the Hamiltonian of the chiral theory is not Hermitian. Bearing in mind the fact that there should be a one-parameter family of higher-spin theories in AdS_4 [77] with the self-dual limits as extremal cases, we expect that there should exist a one-parameter family of higher-spin theories in flat space too. The starting point is to take $Ce^{i\gamma}$ and $\bar{C}e^{-i\gamma}$ as new couplings with C and \bar{C} given by the Metsaev formula. The chiral theories arise in the $e^{\pm i\gamma} \rightarrow \infty$ limits. In particular, there should exist a unitary higher-spin theory, which we are after. In the unitary theory we should have $C^{\lambda_1, \lambda_2, \lambda_3} = \bar{C}^{-\lambda_1, -\lambda_2, -\lambda_3}$, i.e. the theory is parity invariant. Then,

$$[H_3(\mathbb{P}), J_3^{z-}] \sim C\bar{C}\mathbb{P}\bar{\mathbb{P}} \neq 0$$

generates a non-vanishing contribution to (5.1) with $a = z$. This implies that the theory cannot be truncated at the level of cubic vertices and requires higher order interaction terms. This story is very much parallel to the gravity case where the deformation procedure does not stop at the cubic level.

We recall that \mathbf{H}_2 is just an algebraic operator that acts by multiplying by $\sum_i H_2(q_i)$, see (3.49). This means that (5.1) can always be solved formally for j_4^{a-} no matter what are the contributions from other terms. This, however, requires to divide by $\sum_i H_2(q_i)$, which vanishes when all particles go on-shell. The only way to avoid this singularity is to require that the right hand side of (5.1) is proportional to $\sum_i H_2(q_i)$. In other words, we have to solve the equation

$$\left(\mathbf{J}_2^{a-}[h_4] + [H_3, J_3^{a-}] \right) \Big|_{\sum_i H_2(q_i)=0} = 0 \quad (5.2)$$

for h_4 . Once the solution is found, we can simply solve (5.1) for j_4 without producing a singularity.

In the remaining part of this section we solve (5.2) for h_4 in the case of spin zero self-interaction. To do that we make a general local ansatz, expressed in terms of independent variables and fix free

coefficients by requiring that (5.2) is satisfied. As independent variables parametrising dependence of h^4 on transverse momenta we take \mathbb{P}_{12} , \mathbb{P}_{34} , $\bar{\mathbb{P}}_{12}$ and $\bar{\mathbb{P}}_{34}$. Explicitly in the scalar case one has

$$\begin{aligned}
[H_3, J_3^{z-}] = \sum_{\omega=0} \frac{3(-)^\omega \omega}{4} (C^{00\omega})^2 & \left[\frac{\beta_1 - \beta_2}{\beta_1 + \beta_2} \frac{\bar{\mathbb{P}}_{12}^{\omega-1} \mathbb{P}_{34}^\omega}{(\beta_1 + \beta_2)^\omega (\beta_3 + \beta_4)^\omega} + \frac{\beta_3 - \beta_4}{\beta_3 + \beta_4} \frac{\bar{\mathbb{P}}_{34}^{\omega-1} \mathbb{P}_{12}^\omega}{(\beta_1 + \beta_2)^\omega (\beta_3 + \beta_4)^\omega} \right. \\
& + \frac{\beta_1 - \beta_3}{\beta_1 + \beta_3} \frac{\bar{\mathbb{P}}_{13}^{\omega-1} \mathbb{P}_{24}^\omega}{(\beta_1 + \beta_3)^\omega (\beta_2 + \beta_4)^\omega} + \frac{\beta_2 - \beta_4}{\beta_2 + \beta_4} \frac{\bar{\mathbb{P}}_{24}^{\omega-1} \mathbb{P}_{13}^\omega}{(\beta_1 + \beta_3)^\omega (\beta_2 + \beta_4)^\omega} \\
& \left. + \frac{\beta_1 - \beta_4}{\beta_1 + \beta_4} \frac{\bar{\mathbb{P}}_{14}^{\omega-1} \mathbb{P}_{23}^\omega}{(\beta_1 + \beta_4)^\omega (\beta_2 + \beta_3)^\omega} + \frac{\beta_2 - \beta_3}{\beta_2 + \beta_3} \frac{\bar{\mathbb{P}}_{23}^{\omega-1} \mathbb{P}_{14}^\omega}{(\beta_1 + \beta_4)^\omega (\beta_2 + \beta_3)^\omega} \right]. \quad (5.3)
\end{aligned}$$

Taking into account that \mathbf{J}_2^{z-} raises the homogeneity degree in \bar{q} by one and keeps the homogeneity degree in q unchanged, we conclude that, if at all possible, one should be able to solve (5.2) for each ω separately and use the ansatz of the form $h_4^{[\omega]} \sim \mathbb{P}^{\omega-1} \bar{\mathbb{P}}^{\omega-1}$. In other words, ω provides a grading associated with the number of transverse derivatives. Moreover, from lower- ω cases we learned that one can find h_4 's compensating each line of (5.4) separately. Using these observation we found a solution for a general ω to be

$$h_4 = \frac{3}{2} \sum_{\omega} (-)^{\omega+1} (C^{0,0,\omega})^2 h_4^{[\omega]} + (\{1234\} \rightarrow \{1324\}) + (\{1234\} \rightarrow \{1423\}), \quad (5.4)$$

where

$$\begin{aligned}
h_4^\omega = & \frac{-1}{4} \frac{\beta_1 \beta_2 \beta_3 \beta_4}{(\beta_1 + \beta_2)^2 (\beta_3 + \beta_4)^2} s \sum_{n=1}^{\omega-1} \frac{\omega!}{n! (\omega - n)!} \left(\frac{\bar{\mathbb{P}}_{12} \mathbb{P}_{34}}{(\beta_1 + \beta_2)(\beta_3 + \beta_4)} \right)^{n-1} \left(\frac{\mathbb{P}_{12} \bar{\mathbb{P}}_{34}}{(\beta_1 + \beta_2)(\beta_3 + \beta_4)} \right)^{\omega-n-1} \\
& + \sum_{n=0}^{\omega-1} \frac{\omega!}{n! (\omega - n)!} \left(\frac{\bar{\mathbb{P}}_{12} \mathbb{P}_{34} + \mathbb{P}_{12} \bar{\mathbb{P}}_{34}}{(\beta_1 + \beta_2)(\beta_3 + \beta_4)} \right)^n \left(-\frac{1}{4} \frac{\beta_1 - \beta_2}{\beta_1 + \beta_2} \frac{\beta_3 - \beta_4}{\beta_3 + \beta_4} \right)^{\omega-n} s^{\omega-n-1},
\end{aligned}$$

and following [35] we introduced an analog of the s Mandelstam variable:

$$s = \frac{\mathbb{P}_{12} \bar{\mathbb{P}}_{12}}{\beta_1 \beta_2} + \frac{\mathbb{P}_{34} \bar{\mathbb{P}}_{34}}{\beta_3 \beta_4}.$$

This gives the $0 - 0 - 0 - 0$ part of the quartic Hamiltonian of the unitary higher-spin theory. Note that the solution does not have transverse momenta in denominators and should be regarded as perturbatively local.

6 Higher-Spin Equivalence Principle

The Weinberg low-energy theorem [1] implies the conservation of the electric charge and the equivalence principle. However, it is too restrictive for massless higher-spin fields to have long-range interactions, see also [39]. As we already noted in Section 2 the theorem does not formally apply to the light-cone vertices.

In this Section, after discussing few lower spin examples, we will prove that Yang-Mills and gravitational interactions of higher-spin fields exhibit certain universality. In particular, the equivalence principle extends to higher-spin fields as well. The Metsaev solution discussed above obeys the equivalence principle too.

6.1 Examples of Low Spin Fields

What we try to see below is the conditions that arise at the quartic level when some set of cubic vertices is activated, i.e. to probe the holomorphic constraints (4.5) that decouple from H_4 and J_4 , but, as we have seen, can restrict couplings.

Scalar Cubed Theory. This is the simplest and somewhat trivial example:

$$h_3 = \Phi^0 \Phi^0 \Phi^0 [C^{0,0,0} + \text{c.c.}] , \quad J_3 = 0 . \quad (6.1)$$

Thanks to $J_3 = 0$ the commutator $[J_3, H_3]$ vanishes identically revealing that the cubic vertex provides a self-consistent theory and solves (4.4), which is expected, of course.

Yang-Mills theory. For the case of spin-one self-interaction we have to have a colored set of fields since \mathbb{P} is totally anti-symmetric. Therefore, we introduce some anti-symmetric structure constants f^{abc} and let fields carry additional indices too, Φ_a^λ . The cubic vertex reads

$$h_3 = f^{abc} \Phi_a^1 \Phi_b^1 \Phi_c^{-1} \left[\frac{\bar{\mathbb{P}}_{12} C^{1,1,-1} \beta_3}{\beta_1 \beta_2} + \text{c.c.} \right] . \quad (6.2)$$

After summing over cyclic permutations we find that (4.5) is satisfied provided the Jacobi identity for the structure constants is true.

Yang-Mills theory coupled to Scalar Matter. It is also interesting to see how the Yang-Mills fields can couple to matter.¹¹ To this effect we add a one-derivative $0 - 0 - 1$ vertex, where the current built of the scalar fields couples to the Yang-Mills field:

$$h_3 = \left[\frac{\beta_3}{\beta_1 \beta_2} f^{abc} \Phi_a^1 \Phi_b^1 \Phi_c^{-1} \bar{\mathbb{P}}_{12} C^{1,1,-1} + T^{aij} \Phi_a^1 \Phi_i^0 \Phi_j^0 C^{0,0,1} \frac{\bar{\mathbb{P}}_{23}}{\beta_1} + \text{c.c.} \right] . \quad (6.3)$$

Interestingly, after symmetrizing over the permutations (4.5) implies $C^{1,1,-1} = C^{0,0,1}$, i.e. the coupling constants must be equal.

¹¹This example was also discussed in [35, 36], as well as the scalar-tensor theory below.

Pure Gravity. In the case of pure gravity we inject the Einstein-Hilbert two-derivative cubic vertex, i.e. $C^{2,2,-2} \neq 0$, while all other constants are zero:

$$h_3 = \Phi^2 \Phi^2 \Phi^{-2} \left[\frac{\bar{\mathbb{P}}_{12}^2 \beta_3^2 C^{2,2,-2}}{\beta_1^2 \beta_2^2} + \text{c.c.} \right] \quad (6.4)$$

Then the holomorphic part (4.5) of the commutator $[J_3, H_3]$ can be found to identically vanish after symmetrizing over permutations of all four legs, which, at this order, just tells us that gravity might be a consistent theory.

Higher-derivative Gravity. From the covariant approach it is known that one can add a six-derivative R^3 -type vertex, the resulting theory being consistent. In the light-cone approach we start with

$$h_3 = \Phi^2 \Phi^2 \Phi^{-2} \left[\frac{\bar{\mathbb{P}}_{12}^2 \beta_3^2 C^{2,2,-2}}{\beta_1^2 \beta_2^2} \right] + \Phi^2 \Phi^2 \Phi^2 \left[\frac{\bar{\mathbb{P}}_{12}^6 C^{2,2,2}}{\beta_1^2 \beta_2^2 \beta_3^2} \right] + \text{c.c.} \quad (6.5)$$

In the commutator one finds two types of CC terms:

$$(4.5) \sim (...) C^{2,2,-2} C^{2,2,-2} + (...) C^{2,2,-2} C^{2,2,2}, \quad (6.6)$$

which vanish independently after symmetrizing over the four legs. Therefore, the R^3 vertex can be added with an arbitrary coefficient, which is to be expected from the covariant approaches.

Gravity plus Scalar Matter. A different situation is with the scalar-tensor theory, which in addition to gravity contains a two-derivative vertex that couples the scalar field stress-tensor to gravity:

$$h_3 = \Phi^2 \Phi^2 \Phi^{-2} \left[\frac{\bar{\mathbb{P}}_{12}^2 \beta_3^2 C^{2,2,-2}}{\beta_1^2 \beta_2^2} \right] + \Phi^0 \Phi^0 \Phi^2 \left[\frac{\bar{\mathbb{P}}_{12}^2 C^{0,0,2}}{\beta_3^2} \right] + \text{c.c.} \quad (6.7)$$

In this case the vanishing of (4.5) imposes a single constraint:

$$C^{2,2,-2} = C^{0,0,2}, \quad (6.8)$$

i.e. the scalar field coupling equals to that of the gravity — the equivalence principle.

Einstein-Yang-Mills Theory. We can also try to couple a spin-one field to gravity, i.e. to activate the $C^{2,1,-1}$ vertex:

$$h_3 = \Phi^2 \Phi^2 \Phi^{-2} \left[\frac{\bar{\mathbb{P}}_{12}^2 \beta_3^2 C^{2,2,-2}}{\beta_1^2 \beta_2^2} \right] + \Phi^1 \Phi^{-1} \Phi^2 \left[\frac{\bar{\mathbb{P}}_{12}^2 C^{1,-1,2} \beta_2}{\beta_1 \beta_3^2} \right] + \text{c.c.} \quad (6.9)$$

As before the vanishing of (4.5) imposes a single constraint:

$$C^{2,2,-2} = C^{1,-1,2}, \quad (6.10)$$

i.e. the equivalence principle for a Maxwell field.

6.2 Universality of Gravity and Yang-Mills

Even before attempting to look for a complete theory we can ask a simpler question: what happens if we have a higher-spin field which is coupled to gravity or the Yang-Mills theory.

Generalizing the low-spin examples above, we can take a spin- s field and a spin-one Yang-Mills field and turn on $C^{s,-s,1}$ in addition to the Yang-Mills interaction itself. Then, vanishing of the holomorphic terms in $[H_3, J_3]$ implies that all higher-spin fields couple universally to spin-one:

$$s - s - 1 : \quad C^{s,-s,1} = C^{1,1,-1} = g.$$

The same exercise for the gravitation interaction, i.e. with $C^{2,2,-2}$ and $C^{2,s,-s}$ switched on implies that all higher-spin fields couple universally to spin-two:¹²

$$s - s - 2 : \quad C^{s,-s,2} = C^{2,2,-2} = g l_p.$$

The fact that the strength of the backreaction from higher-spin fields on gravity must be the same for all spins $s = 0, 1, 2, 3, 4, \dots$ is a reincarnation of the equivalence principle which, as it turns out, holds true for fields of any spin.¹³

The higher-spin equivalence principle also implies that there is a system made of graviton and a spin- s field with only the Einstein-Hilbert $C^{2,2,-2}$ and gravitational $C^{s,-s,2}$ vertices switched on that solves the holomorphic constraints (4.5). Therefore, this solution explicitly avoids the Aragone-Deser argument in the light-cone approach and suggests that it may be possible to put higher-spin fields on more general backgrounds. However, (4.5) is a necessary condition and an obstruction can come from the rest of the constraints (4.6) and higher orders.

It should be noted that the Weinberg low-energy theorem, if applied literally to the higher-spin case, does imply that all couplings should be equal but it simultaneously imposes a too restrictive conservation law that can only be obeyed by the scattering processes that simply permute the particles' momenta. Pessimistically, this should then be seen later in the light-cone approach too. Optimistically, the Weinberg theorem can be avoided by the light-cone approach.

¹²Technically, what one does is to take h_3 with $C^{2,2,-2}$, $C^{2,s,-s}$ vertices and then to symmetrize over the fields in (4.5). The outcome is proportional to a complicated kinematical factor and $C^{2,s,-s}(C^{2,2,-2} - C^{2,s,-s})$, which leads to the result and similarly in the case of the Yang-Mills interaction.

¹³We do not consider fermionic higher-spin fields in this paper, but undoubtedly they have to follow the same law.

7 Conclusions and Discussion

We pointed out that due to the holomorphic splitting of the Poincare algebra consistency relations there exists a complete chiral higher-spin theory in $4d$ flat space. Such a theory provides a counterexample to a widespread belief that higher-spin interactions are impossible in the Minkowski space. However, the theory is non-unitary.

While the chiral theory is an encouraging result, we expect the unitary higher-spin theory to exist too. Its derivation requires more efforts since the Poincare deformation procedure does not stop at the cubic order. We have fixed a part of the quartic Hamiltonian that determines an infinite series of the quartic contact vertices of the scalar field. This can be thought of as the Minkowski space counterpart of the AdS result obtained recently in [37, 38]. In particular the flat space quartic action shares some features with its AdS_4 cousin: it is naively non-local in having an unbounded order in derivatives arranged into a series of positive powers of the transverse momenta. However, there are no wild non-localities of type $1/\square$ or $1/p_i \cdot p_j$, which would trivialize the deformation procedure [78]. Such non-localities arise in some of the covariant studies [53], but not in the others [73]. Formally, the quartic scalar self-interaction drops off the Noether procedure at this order since scalar field does not feature its own gauge parameter. The equation for the quartic scalar vertex is a part of the quintic Noether consistency conditions.

The mild non-localities we observed are to be expected since higher-spin theories are not power-counting renormalizable and coupling conspiracy is the only way for them to be quantum consistent, i.e. to have an infinitely-many of Slavnov-Taylor identities as a result of a clever fine-tuning of higher-derivative interactions. The light-cone locality requirement is not to have transverse momenta in denominators, otherwise any deformation can be formally extended to higher-orders [78]. The quartic $0 - 0 - 0 - 0$ coupling we found is local in this sense.

One of the surprises of the light-cone approach as compared to covariant methods is the existence of an additional, exceptional, series of cubic vertices which contains the two-derivative gravitational interactions of higher-spin fields. These vertices are also seen by the amplitude methods.

As was observed by Metsaev, at the quartic order the Poincare algebra consistency relations split into the three parts, two of which do not involve the quartic generators at all but do impose restrictions on possible couplings. Some mild assumptions on the spectrum of a theory are needed, otherwise there are infinitely many solutions, some of which might be of interest too in the context of conformal higher-spin fields, [79], or gravitational interactions. We gave a simple derivation of the solution found by Metsaev in Appendix C.

The latter was impossible to see by covariant methods for the two reasons: (i) the holomorphic decomposition of the Poincare algebra consistency relations is essentially not Lorentz covariant; (ii) the solution we are interested in requires the exceptional vertices to be present. Therefore, at least for some of the problems the covariant methods turn out to be too restrictive (or at least an appropriate set of auxiliary or compensator fields is required and still unknown). Let us note that this solution rules out indirectly consistent higher-spin theories in flat space that are manifestly Lorentz covariant.

Here it is worth stressing that any QFT resulting from the light-cone approach is Poincare invariant by definition.

These results are the $4d$ Minkowski counterparts of the higher-spin algebra uniqueness theorems [42–46, 48] proved recently in the context of AdS/CFT, which imply the same statement for massless higher-spin fields in *AdS* or higher-rank conserved tensors in the dual CFT picture. The latter results are even more restrictive because the higher-spin algebras can be shown to be generated by free conformal fields.

A more general idea that we would like to pursue is to establish a relation between higher-spin theories in flat and AdS spaces. In particular, it would be interesting to see if there exists a flat limit in some sense (naive limit of vanishing cosmological constant should not work beyond the cubic level [52]). It is tempting to propose that the light-cone approach is a suitable framework for such a limit to be smooth. Indeed, one finds many similarities between higher spins in flat and AdS spaces to support this idea.

Firstly, it is the vanishing of the scalar self-coupling, which is a sensible interaction. It comes as a surprise in flat space. We would like to stress that this is consistent with the *AdS*-lift of this theory where the absence of ϕ^3 is required by the *AdS/CFT* correspondence [9, 11, 12]. In the critical vector model $\langle \sigma \sigma \sigma \rangle|_{d=3} = 0$ and therefore the bulk coupling is expected to be zero at $d = 4$. Meanwhile, in the free vector model $\langle \varphi^2 \varphi^2 \varphi^2 \rangle \neq 0$, but the bulk vertex is extremal, [11, 12], therefore the bulk coupling should approach zero near four-dimensions, and indeed it does [37].

Secondly, in flat space the cubic action is given by the simple Metsaev solution $\Gamma(\lambda_1 + \lambda_2 + \lambda_3)^{-1}$. Later [37] the same pattern was observed for $s - 0 - 0$ vertices in *AdS*₄ higher-spin theory and conjectured [80] to be the same for all $s_1 - s_2 - s_3$ with the explicit proof given in [81] in the course of reconstructing the complete cubic action of the Type-A higher-spin theory. A new piece of evidence may come from the twistorial approach to conformal higher-spin theory [82], if the three-point functions of the unitary truncation turn out to be the same.

Thirdly, both in AdS and flat cases we see the option of having a consistent parity-violating theory, whose extreme limit is the chiral theory presented in the paper. This is just an observation in flat space, while the AdS counterpart is well supported by the existence of Chern-Simons matter theories [77]. It would be interesting to construct the chiral higher-spin theory in AdS too, which is supposed to terminate at the cubic level contrary to the unitary higher-spin theories. In this regard, it is worth mentioning that there is a one-parameter family of boundary conditions [83]:

$$e^{+i\gamma}C = e^{-i\gamma}\bar{C}, \quad e^{i\gamma} = \sqrt{\frac{k + iN}{k - iN}}, \quad (7.1)$$

where k is the Chern-Simons level and N is the number of matter fields. The two standard limits are $C = \pm\bar{C}$ and correspond to ordinary and alternate boundary conditions. It is interesting that there are two extremal cases where $e^{\pm i\gamma}$ goes to infinity and therefore imposes $C = 0$ or $\bar{C} = 0$. Clearly, in the bulk such a limit results in a self-dual higher-spin theory, while its interpretation from the CFT side is unclear [84]. The simplicity of the self-dual *AdS*₄ higher-spin theory that we expect is based

on its flat space cousin.

Another fruitful direction to go is to extend the $4d$ quartic results to higher-dimensions. In particular, it is interesting to see if there exists a phenomenon similar to the holomorphic factorization of the Poincare algebra at the quartic order that allows to fix the cubic action before encountering any problems at the quartic level. Lower dimensions $d = 5, \dots$ should be of more interest due to the specific structures on the Wigner little group. In particular in $d = 5$ the relevant algebra is $su(2)$ and therefore the spinning degrees of freedom should be governed by $hs(\lambda)$ [85, 86] which is familiar from the $3d$ higher-spin studies [58–60]. The case of AdS_5/CFT_4 higher-spin duality can be richer owing to the existence of doubletons [87, 88], i.e. massless conformal fields of arbitrary spin. For any $j = 0, \frac{1}{2}, 1, \frac{3}{2}, \dots$ there should exist a higher-spin dual, called Type-A,B,C,... that computes the correlators of the primaries that are bilinear in spin- j doubletons. This should work classically, while there can be some obstructions at the quantum level for $j > 1$ [89], see also [90–92] for the details of the approach. The relevant one-parameter family of higher-spin algebras was discussed in [93–98], while the relation to $hs(\lambda)$ is manifest in the quasiconformal approach of [94, 96].

Another direction along the lines of recent studies [99] is to try to solve the quartic consistency relations for a stringy spectrum of fields, i.e. instead of massless higher-spin fields one can try to add massive higher-spins fields and to see what are the options for the multiplets that are consistent with the Poincare algebra. It is not difficult to see that the presence of an at least one massive higher-spin field will require the spin in the multiplet to be unbounded from above as in the massless case. The detailed classification of such multiplets is absent.

Lastly, we attempted to construct the unitary higher-spin theory in flat space. Even though we found only the quartic scalar self-interaction and not the full quartic Hamiltonian we at least have not faced any obstructions. Moreover, in the $4d$ light-cone approach higher-spin fields are not that different from the scalar one. In this respect our result gives us hope that reconstruction of the full quartic Hamiltonian is also possible. On another hand, the no-go results, especially those that were obtained within the BCFW approach that is closer to the light-cone one as compared to covariant methods, still suggest that the light-cone analysis can face certain difficulties as well. It would be interesting to establish this in future.

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A Most General Form of Cubic Vertices

We write the most general ansatz that solves all the kinematical constraints, i.e. have the correct homogeneity in \mathbb{P} , $\bar{\mathbb{P}}$ and β ’s:

$$h_{\lambda_1, \lambda_2, \lambda_3} = C^{\lambda_1, \lambda_2, \lambda_3} \frac{\bar{\mathbb{P}}^{\lambda_1 + \lambda_2 + \lambda_3}}{\beta_1^{\lambda_1} \beta_2^{\lambda_2} \beta_3^{\lambda_3}} F \left[\frac{\mathbb{P} \bar{\mathbb{P}}}{\beta_1 \beta_2}, \frac{\beta_1}{\beta_2} \right], \quad (\text{A.1})$$

where $F[x, y]$ is a priori an arbitrary function of two arguments. Also we solved explicitly for the momenta conservation, so that β_3 is unnecessary. Applying J_2 we find:

$$\frac{1}{h} \sum J_2^T h = \frac{1}{F} \frac{\mathbb{P} \left(O[F] - 2 \frac{\Lambda}{\beta_2} F(x, y) \right)}{3 \beta_2^2 y (y + 1)}, \quad (\text{A.2})$$

$$\frac{1}{h} \sum \bar{J}_2^T h = \frac{1}{F} \frac{\bar{\mathbb{P}}}{3 \beta_2^2 y (y + 1)} O[F], \quad (\text{A.3})$$

$$O[F] = -3y(y + 1) F^{(0,1)}(x, y) - x(y - 1) F^{(1,0)}(x, y), \quad (\text{A.4})$$

where in the last line we defined the differential operator O that contributes both to $J_2 h_3$ and $\bar{J}_2 h_3$. It has a zero mode that is responsible for the field redefinitions: $f(\frac{x^3 y}{(y+1)^2})$. Therefore, it is convenient to rewrite the ansatz as

$$h_{\lambda_1, \lambda_2, \lambda_3} = C^{\lambda_1, \lambda_2, \lambda_3} \frac{\bar{\mathbb{P}}^{\lambda_1 + \lambda_2 + \lambda_3}}{\beta_1^{\lambda_1} \beta_2^{\lambda_2} \beta_3^{\lambda_3}} F \left[\frac{\mathbb{P} \bar{\mathbb{P}}}{(\beta_1 \beta_2 \beta_3)^{2/3}}, \frac{\beta_1}{\beta_2} \right]. \quad (\text{A.5})$$

We should not worry about fractional powers. Whenever needed they can always be compensated by the y -dependence. Now the operators simplify a bit (simple derivative remains)

$$\frac{1}{h} \sum J_2^T h = \frac{1}{F} \frac{\mathbb{P} \left(-3y(y + 1) F^{(0,1)}(x, y) - 2 \frac{\Lambda}{\beta_2} F(x, y) \right)}{3 \beta_2^2 y (y + 1)}, \quad (\text{A.6})$$

$$\frac{1}{h} \sum \bar{J}_2^T h = \frac{1}{F} \frac{-\bar{\mathbb{P}} F^{(0,1)}(x, y)}{\beta_2^2}. \quad (\text{A.7})$$

This provides the most general solution for the generators at the cubic level:

$$j_3 = \left[\frac{\mathbb{P}\bar{\mathbb{P}}}{\beta_1\beta_2\beta_3} \right]^{-1} \sum J_2^T h_3, \quad \bar{j}_3 = \left[\frac{\mathbb{P}\bar{\mathbb{P}}}{\beta_1\beta_2\beta_3} \right]^{-1} \sum \bar{J}_2^T h_3. \quad (\text{A.8})$$

The redefinitions correspond to adding a multiple of $H_2 \sim \mathbb{P}\bar{\mathbb{P}}$. Therefore, the solutions can be made purely (anti)-holomorphic in \mathbb{P} and $\bar{\mathbb{P}}$, which is the choice made by Metsaev and quoted in the main text.

B Triviality of $[J, J] = 0$

The last and the most difficult part of the commutation relations $[J^{a-}, J^{b-}] = 0$ is always true¹⁴ provided J^{a-} is solved for in terms of H at a given order. Indeed, at order n and with all kinematical constraints already solved for we have two equations:

$$[J_2^{a-}, H_n] + \sum_{i+j=n; i,j>1} [J_i^{a-}, H_j] = [H_2, J_n^{a-}], \quad (\text{B.1})$$

$$[J_2^{a-}, J_n^{b-}] - (ab) + \sum_{i+j=n; i,j>1} [J_i^{a-}, J_j^{b-}] = 0. \quad (\text{B.2})$$

We note that the action of H_2 in the first equation is always algebraic and is therefore invertible off-shell, i.e. outside the zero-energy surface $\sum E_i = 0$. Let us start at the cubic order where we have

$$[J_2^{a-}, H_3] = [H_2, J_3^{a-}], \quad [J_2^{a-}, J_3^{b-}] - (ab) = 0. \quad (\text{B.3})$$

In order to check that the last equality is automatically true we use the invertibility of $[H_2, \bullet]$:

$$[J_2^{a-}, J_3^{b-}] - (ab) = 0, \quad \Longleftrightarrow \quad [H_2, [J_2^{a-}, J_3^{b-}]] - (ab) = 0. \quad (\text{B.4})$$

Using the Jacobi identity and $[H_2, J_2^{a-}] = 0$ we have the following chain of implications

$$[H_2, [J_2^{a-}, J_3^{b-}]] - (ab) = [J_2^{a-}, [H_2, J_3^{b-}]] - (ab) = \quad (\text{B.5})$$

$$= [J_2^{a-}, [J_2^{b-}, H_3]] - (ab) = -[H_3, [J_2^{a-}, J_2^{b-}]] = 0, \quad (\text{B.6})$$

¹⁴We are indebted to Ruslan Metsaev for claiming that this fact should be true.

where in the last step we used the algebra relations at one order less. At the quartic order we find

$$[H_2, [J_2^a, J_4^b]] - (ab) + [H_2, [J_3^a, J_3^b]] = \quad (\text{B.7})$$

$$[J_2^a, [H_2, J_4^b]] - [J_3^a, [H_2, J_3^b]] - (ab) = \quad (\text{B.8})$$

$$[J_2^a, [J_2^b, H_4]] + [J_2^a, [J_3^b, H_3]] - [J_3^a, [H_2, J_3^b]] - (ab) = \quad (\text{B.9})$$

$$0 - [H_3, [J_2^a, J_3^b]] - [J_3^a, [J_2^b, H_3]] - (ab) = [H_3, [J_3^a, J_2^b]] - (ab) = 0 \quad (\text{B.10})$$

where we used several times the cubic order relations $[J_2, J_3] = 0$ and $[H_2, J_3] = [J_2, H_3]$. The general proof follows the same logic, but is a bit boring.

C Metsaev Solution

First step is to evaluate explicitly the commutator in (4.5), which results in:

$$\sum_{\omega} \text{Sym} \left[\frac{(\lambda_1 + \omega - \lambda_2)\beta_1 - (\lambda_2 + \omega - \lambda_1)\beta_2}{\beta_1 + \beta_2} C^{\lambda_1, \lambda_2, \omega} C^{\lambda_3, \lambda_4, -\omega} \bar{\mathbb{P}}_{12}^{\lambda_1 + \lambda_2 + \omega - 1} \bar{\mathbb{P}}_{34}^{\lambda_3 + \lambda_4 - \omega} \right] = 0, \quad (\text{C.1})$$

where Sym is a complete symmetrisation, which originates from contraction with $\Phi_{qi}^{\lambda_i}$. This symmetrisation is essential: if it had been omitted, the solution (4.7) would have been lost. The expression appearing in brackets in (C.1) is manifestly symmetric with respect to permutations $1 \leftrightarrow 2$ and $3 \leftrightarrow 4$. To achieve complete symmetry one has to add five other non-trivial permutations

$$6 \cdot \{1, 2, 3, 4\} \rightarrow \{1, 2, 3, 4\} + \{1, 3, 2, 4\} + \{1, 4, 2, 3\} + \{3, 4, 1, 2\} + \{2, 4, 1, 3\} + \{2, 3, 1, 4\}.$$

Provided momentum conservation is taken into account, there are five independent variables among $\bar{\mathbb{P}}_{ij}$ and β_i . So, generically, to solve an equation of the form (C.1) one would need to express the left hand-side in terms of five independent variables and then solve it for all values of these variables. It, however, turns out, that the left hand side of (C.1) can be expressed in terms of $\bar{\mathbb{P}}_{ij}$ only, among which only three are independent. This can be seen if we group the term in brackets in (C.1) and the one obtained by the permutation $\{1, 2, 3, 4\} \rightarrow \{3, 4, 1, 2\}$ and relabeling $\omega \rightarrow -\omega$. Summing them, we find that the β -dependence cancels

$$\begin{aligned} & \frac{(\lambda_1 + \omega - \lambda_2)\beta_1 - (\lambda_2 + \omega - \lambda_1)\beta_2}{\beta_1 + \beta_2} \bar{\mathbb{P}}_{34} + \frac{(\lambda_3 - \omega - \lambda_4)\beta_3 - (\lambda_4 - \omega - \lambda_3)\beta_4}{\beta_3 + \beta_4} \bar{\mathbb{P}}_{12} \\ &= (\lambda_1 - \lambda_2)\bar{\mathbb{P}}_{34} + (\lambda_3 - \lambda_4)\bar{\mathbb{P}}_{12} + \frac{\omega}{2}(\bar{\mathbb{P}}_{13} - \bar{\mathbb{P}}_{23} + \bar{\mathbb{P}}_{24} - \bar{\mathbb{P}}_{14}). \end{aligned} \quad (\text{C.2})$$

In terms of independent variables (4.14), equation (C.1) can be rewritten in a more suggestive form

$$\begin{aligned} & \sum_{\omega} \left[((\lambda_1 - \lambda_2 + \lambda_3 - \lambda_4)A + (\lambda_1 - \lambda_2 - \lambda_3 + \lambda_4)B - 2\omega C) \right. \\ & \quad \left. C^{\lambda_1, \lambda_2, \omega} C^{\lambda_3, \lambda_4, -\omega} (A - B)^{\lambda_1 + \lambda_2 + \omega - 1} (A + B)^{\lambda_3 + \lambda_4 - \omega - 1} \right] \end{aligned}$$

$$\begin{aligned}
& +((- \lambda_1 + \lambda_3 + \lambda_2 - \lambda_4)B + (- \lambda_1 + \lambda_3 - \lambda_2 + \lambda_4)C + 2\omega A) \\
& \quad C^{\lambda_1, \lambda_3, \omega} C^{\lambda_2, \lambda_4, -\omega} (B - C)^{\lambda_1 + \lambda_3 + \omega - 1} (-B - C)^{\lambda_2 + \lambda_4 - \omega - 1} \\
& +((\lambda_1 - \lambda_4 - \lambda_2 + \lambda_3)A + (\lambda_1 - \lambda_4 + \lambda_2 - \lambda_3)C - 2\omega B) \\
& \quad C^{\lambda_1, \lambda_4, \omega} C^{\lambda_2, \lambda_3, -\omega} (C - A)^{\lambda_1 + \lambda_4 + \omega - 1} (A + C)^{\lambda_2 + \lambda_3 - \omega - 1} \Big] = 0. \quad (C.3)
\end{aligned}$$

For $C = 0$ it gives

$$(A - \mu B)f(A, B) = k_{\Lambda-1}A^{\Lambda-1} + k_{\Lambda-2}A^{\Lambda-2}B + k_1AB^{\Lambda-2} + k_0B^{\Lambda-1}, \quad (C.4)$$

where we denoted

$$\begin{aligned}
f(A, B) & \equiv \sum_{\omega} C^{\lambda_1, \lambda_2, \omega} C^{\lambda_3, \lambda_4, -\omega} (A - B)^{\lambda_1 + \lambda_2 + \omega - 1} (A + B)^{\lambda_3 + \lambda_4 - \omega - 1}, \quad (C.5) \\
\mu & \equiv -\frac{\lambda_1 - \lambda_2 - \lambda_3 + \lambda_4}{\lambda_1 - \lambda_2 + \lambda_3 - \lambda_4}, \quad \Lambda \equiv \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4,
\end{aligned}$$

and $k_{\Lambda-1}, \dots, k_0$ can be extracted from (C.3), but will not be needed for the following discussion.

Next we note: (i) from all possible terms of the form $A^m B^{\Lambda-m-1}$ the right hand side of (C.4) contains only four; (ii) $A - \mu B$ should be a divisor of the right hand side; (iii) cubic vertices with the total helicity being odd vanish, so $f(A, B) = -f(B, A)$.

These three conditions allow to fix $f(A, B)$ up to an overall factor. Indeed, the requirement that $A - \mu B$ is a divisor of the right hand side allows to express

$$k_0 = -k_{\Lambda-1}\mu^{\Lambda-1} - k_{\Lambda-2}\mu^{\Lambda-2} - k_1\mu.$$

Then we find

$$\begin{aligned}
f(A, B) & = k_{\Lambda-1}A^{\Lambda-2} + (k_{\Lambda-1}\mu + k_{\Lambda-2})A^{\Lambda-3}B + (k_{\Lambda-1}\mu^2 + k_{\Lambda-2}\mu)A^{\Lambda-4}B^2 + \dots, \\
& + (k_{\Lambda-1}\mu^{\Lambda-3} + k_{\Lambda-2}\mu^{\Lambda-4})B^{\Lambda-3}A + (k_1 + k_{\Lambda-1}\mu^{\Lambda-2} + k_{\Lambda-2}\mu^{\Lambda-3})B^{\Lambda-2}. \quad (C.6)
\end{aligned}$$

Employing $f(A, B) = -f(B, A)$ we get, in particular,

$$\begin{aligned}
k_{\Lambda-1} & = -(k_1 + k_{\Lambda-1}\mu^{\Lambda-2} + k_{\Lambda-2}\mu^{\Lambda-3}), \\
k_{\Lambda-1}\mu + k_{\Lambda-2} & = -(k_{\Lambda-1}\mu^{\Lambda-3} + k_{\Lambda-2}\mu^{\Lambda-4}).
\end{aligned}$$

For real μ this implies $k_{\Lambda-1}\mu + k_{\Lambda-2} = 0$ and $k_1 = -k_{\Lambda-1}$. Hence,

$$f(A, B) = k_{\Lambda-1}(A^{\Lambda-2} - B^{\Lambda-2}). \quad (C.7)$$

Then, (C.5) leads to

$$C^{\lambda_1, \lambda_2, \omega} C^{\lambda_3, \lambda_4, -\omega} = \frac{X(\lambda_1, \lambda_2, \lambda_3, \lambda_4)}{(\lambda_1 + \lambda_2 + \omega - 1)!(\lambda_3 + \lambda_4 - \omega - 1)!}, \quad (\text{C.8})$$

where $X(\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ remains to be specified. To fix X , let us divide (C.8) by the same equation with $\lambda_3 \rightarrow \lambda_4$ and $\lambda_4 \rightarrow \lambda_6$

$$\frac{C^{\lambda_3, \lambda_4, -\omega}}{C^{\lambda_5, \lambda_6, -\omega}} = \frac{X(\lambda_1, \lambda_2, \lambda_3, \lambda_4) (\lambda_5 + \lambda_6 - \omega - 1)!}{X(\lambda_1, \lambda_2, \lambda_5, \lambda_6) (\lambda_3 + \lambda_4 - \omega - 1)!}.$$

This implies that X factorises

$$X(\lambda_1, \lambda_2, \lambda_3, \lambda_4) = Y(\lambda_1, \lambda_2) Y(\lambda_3, \lambda_4).$$

Plugging this back into (C.8) we find

$$\frac{C^{\lambda_1, \lambda_2, \omega}}{Y(\lambda_1, \lambda_2)} (\lambda_1 + \lambda_2 + \omega - 1)! = Z(\omega), \quad (\text{C.9})$$

where $Z(\omega)$ is another unknown function. Using the symmetry of $C^{\lambda_1, \lambda_2, \omega}$ we obtain

$$Y(\lambda_1, \lambda_2) = W Z(\lambda_1) Z(\lambda_2)$$

and hence

$$C^{\lambda_1, \lambda_2, \omega} = W \frac{Z(\lambda_1) Z(\lambda_2) Z(\omega)}{(\lambda_1 + \lambda_2 + \omega - 1)!}.$$

Substituting this again in (C.8) we get

$$Z(\omega) \cdot Z(-\omega) = 1,$$

which is equivalent to

$$Z(\omega) = e^{\sigma(\omega)}, \quad \sigma(-\omega) = -\sigma(\omega).$$

Eventually, we obtain that

$$C^{\lambda_1, \lambda_2, \lambda_3} = W \cdot \frac{e^{\sigma(\lambda_1) + \sigma(\lambda_2) + \sigma(\lambda_3)}}{(\lambda_1 + \lambda_2 + \lambda_3 - 1)!}, \quad (\text{C.10})$$

where $\sigma(\lambda)$ is an arbitrary odd function. Substituting this into (C.3) we find no further constraints on W and $\sigma(\lambda)$. So, (C.10) provides a general solution of the consistency condition (4.5). For $\sigma(\lambda) = \lambda \cdot \ln K$ we reproduce the solution of Metsaev. Our formula (C.10) provides its rather obvious generalisation, where each spin λ has its own coupling constant $\exp \sigma(\lambda)$, but they can be eaten up by rescaling the fields.

All arguments of the derivation above apply if we assume that all three spins entering each vertex are even. Also there is a similar system for the \bar{C} coefficients.

Let us consider one more solution of the holomorphic constraints (4.4). Usually, the higher-spin fields are dressed by Yang-Mills groups in a stringy Chan-Paton way [36, 100]. Let us instead assume that all fields take values in the adjoint of some Lie algebra of internal symmetry with structure constants f^{a_1, a_2, a_3} . Therefore, we should keep the space-time part of cubic vertices unchanged and multiply the coupling constants by f^{a_1, a_2, a_3}

$$C^{\lambda_1, \lambda_2, \lambda_3} \rightarrow C^{\lambda_1, \lambda_2, \lambda_3} f^{a_1, a_2, a_3}. \quad (\text{C.11})$$

Since the structure constants are totally anti-symmetric, this changes the symmetry of the vertices to the opposite one. Namely, the vertices with the total spin being odd are totally symmetric, while the vertices with the total spin being even effectively vanish.

Proceeding along the same lines as before we obtain a consistency condition which differs from (C.3) by replacement (C.11). The consistency condition should hold as a consequence of the Jacobi identity. This implies that one has to demand

$$\begin{aligned} \sum_{\omega} \Big[& ((\lambda_1 - \lambda_2 + \lambda_3 - \lambda_4)A + (\lambda_1 - \lambda_2 - \lambda_3 + \lambda_4)B - 2\omega C) \\ & C^{\lambda_1, \lambda_2, \omega} C^{\lambda_3, \lambda_4, -\omega} (A - B)^{\lambda_1 + \lambda_2 + \omega - 1} (A + B)^{\lambda_3 + \lambda_4 - \omega - 1} \\ & + ((-\lambda_1 + \lambda_3 + \lambda_2 - \lambda_4)B + (-\lambda_1 + \lambda_3 - \lambda_2 + \lambda_4)C + 2\omega A) \\ & C^{\lambda_1, \lambda_3, \omega} C^{\lambda_2, \lambda_4, -\omega} (B - C)^{\lambda_1 + \lambda_3 + \omega - 1} (-B - C)^{\lambda_2 + \lambda_4 - \omega - 1} \Big] = 0. \end{aligned} \quad (\text{C.12})$$

Setting $C = 0$ we find that

$$(A - \mu B)f(A, B) = k_1 AB^{\Lambda - 2} + k_0 B^{\Lambda - 1}, \quad (\text{C.13})$$

where $f(A, B)$, μ and Λ were defined in (C.5). By requiring that $(A - \mu B)$ is a divisor of the right hand side we find

$$f(A, B) = k_0 B^{\Lambda - 2}. \quad (\text{C.14})$$

Due to the fact that the vertices with the total even spin vanish we have $f(A, B) = f(B, A)$. This symmetry property is compatible with (C.14) only if $\Lambda = 2$. This implies

$$C^{\lambda_1, \lambda_2, \omega} C^{\lambda_3, \lambda_4, -\omega} = X(\lambda_1, \lambda_2, \lambda_3, \lambda_4) \delta(\lambda_1 + \lambda_2 + \omega - 1) \delta(\lambda_3 + \lambda_4 - \omega - 1). \quad (\text{C.15})$$

Proceeding as in the case of no internal symmetry, we obtain the solution

$$C^{\lambda_1, \lambda_2, \lambda_3} = W \cdot e^{\sigma(\lambda_1) + \sigma(\lambda_2) + \sigma(\lambda_3)} \delta(\lambda_1 + \lambda_2 + \lambda_3 - 1). \quad (\text{C.16})$$

All arguments of the derivation above apply if we assume that all three spins entering each vertex are odd. In the context of Chan-Paton dressing of higher-spin fields this solution was found by Metsaev

in [79].

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